Mark scheme

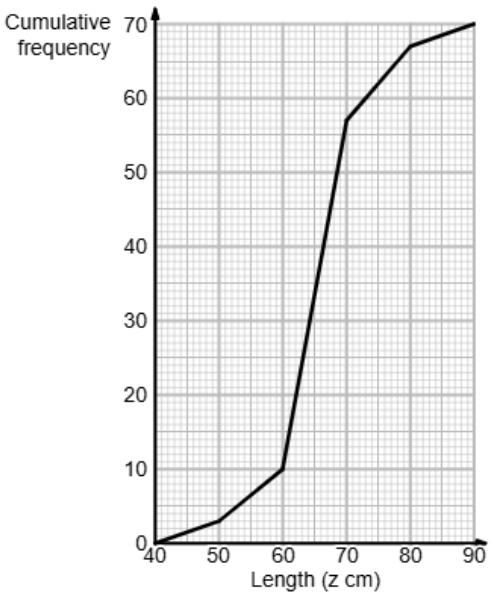
GCSE Edexcel Higher Tier Paper 2 & 3 Likely Topics - 2025

Question 1

Time (y seconds)	Cumulative Frequency
$50 < y \leq 60$	6
$50 < y \leq 70$	13
$50 < y \leq 80$	36
$50 < y \leq 90$	48
$50 < y \leq 100$	55
$50 < y \leq 110$	70

Time (y seconds)	Cumulative frequency
$\mathbf{50 < y \leq 60}$	6
$\mathbf{50 < y \leq 70}$	13
$\mathbf{50 < y \leq 80}$	36
$\mathbf{50 < y \leq 90}$	48
$\mathbf{50 < y \leq 100}$	55
$\mathbf{50 < y \leq 110}$	70

Question 2



There is no data below 40 so you can plot this with a cumulative frequency of 0.

You then need to plot the end of each class interval with its cumulative frequency.

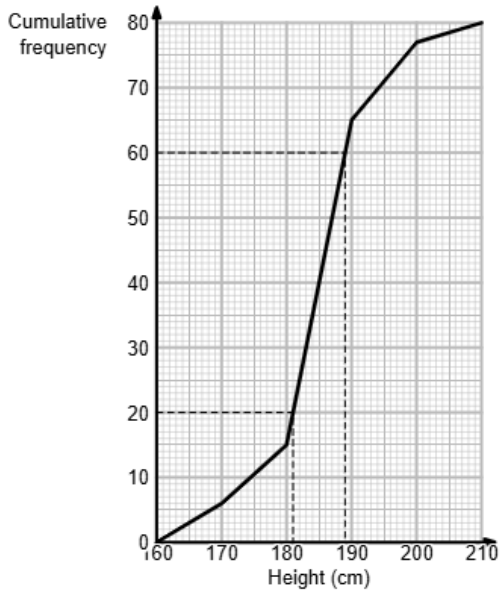
Question 3

① Find the position of the quartiles.

$\frac{80}{4} = 20$

$\frac{80}{4} \times 3 = 60$

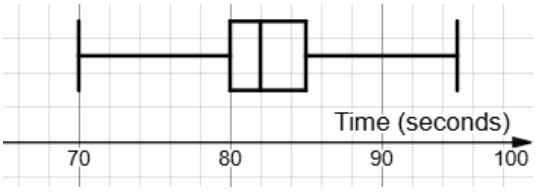
② Find the points on the graph where the cumulative frequency is 20 and 60.



③ Conclude.

The IQR is $189 - 181 = 8$

Question 4



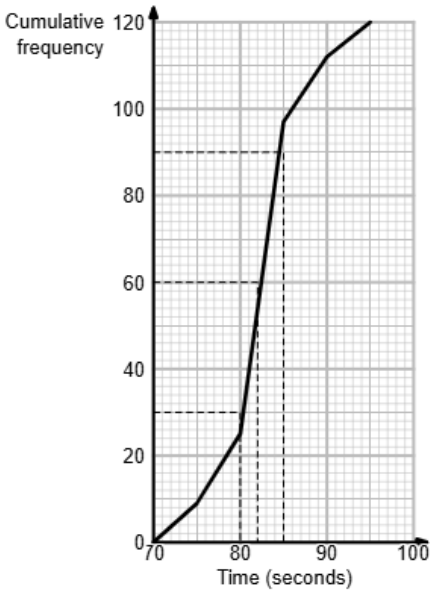
① Find the position of the median and quartiles.

Lower quartile: $\frac{120}{4} = 30$

Median: $\frac{120}{2} = 60$

Upper quartile: $\frac{120}{4} \times 3 = 90$

② Find the points on the graph where the cumulative frequency is 30, 60 and 90.



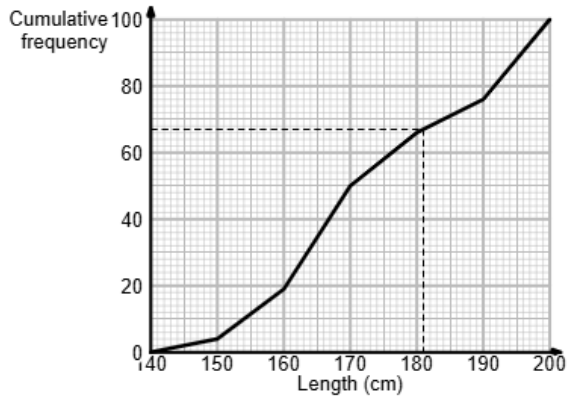
③ Conclude.

Minimum	70
Lower Quartile	80
Median	82
Upper Quartile	85
Maximum	95

Question 5

animals

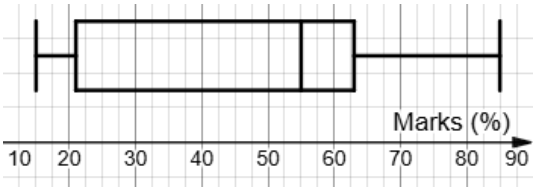
① Find the point on the graph where the length is 181 cm.

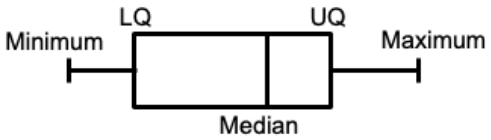


③ Conclude.

The answer is 67.

Question 6





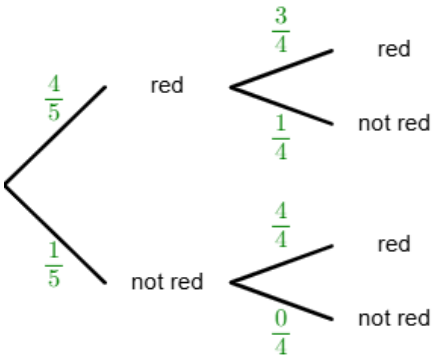
Question 7

$a = \frac{4}{5}, b = \frac{1}{5}, c = \frac{3}{4}, d = \frac{1}{4}, e = \frac{4}{4}, f = \frac{0}{4}$

There are $b = \frac{1}{5}$ red balls out of $b = \frac{1}{5}$ balls therefore $a = \frac{4}{5}$

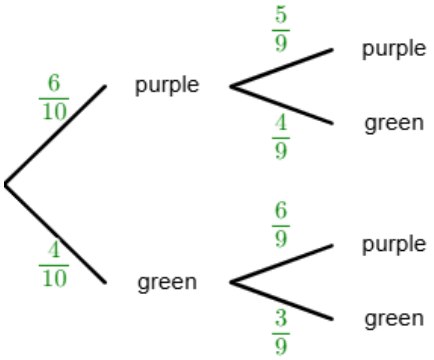
There are 1 balls that are not red out of $b = \frac{1}{5}$ balls therefore $b = \frac{1}{5}$

As Christine keeps the red ball, the total number of ball decreases by one the second time she picks a ball.



Question 8

$\frac{48}{90}$

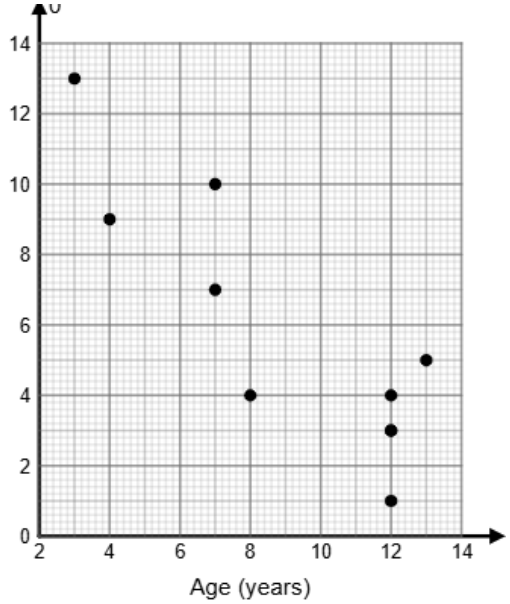


purple then green: $\frac{6}{10} \times \frac{4}{9} = \frac{24}{90}$

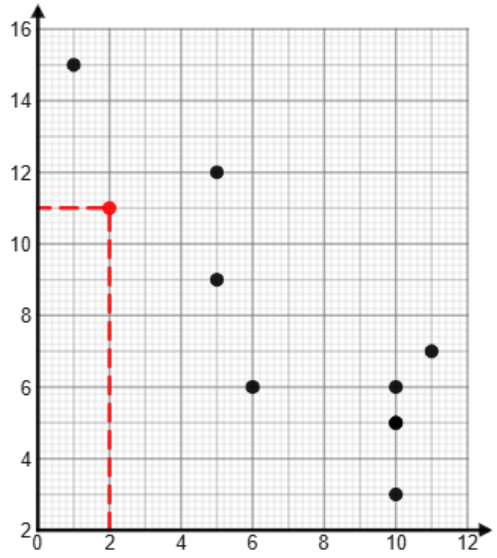
green then purple: $\frac{4}{10} \times \frac{6}{9} = \frac{24}{90}$

$\frac{24}{90} + \frac{24}{90} = \frac{48}{90}$

Question 9



① Plot the point $(2,11)$ by going x units along the x-axis and 11 units along on the y-axis.



Question 10

$(\ 1\ ,15\ )$

① Identify the coordinates for each student.

$(1,15), (11,5), (18,3), (26,20)$

The x-coordinate will be the March score and the y-coordinate will be the April score.

② Calculate the improvement.

$(1,15) \rightarrow 15 - 1 = 14$
$(11,5) \rightarrow 5 - 11 = -6$
$(18,3) \rightarrow 3 - 18 = -15$
$(26,20) \rightarrow 20 - 26 = -6$

③ Find the greatest improvement.

The coordinates of the greatest improvement are $(1,15)$.

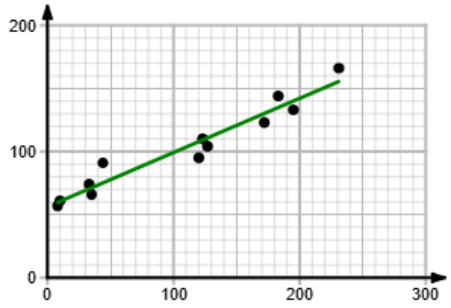
Question 11

strong positive correlation

① Identity the type of correlation.

An upward trend in the data means it is **positive correlation**.

② Identify the strength of correlation.

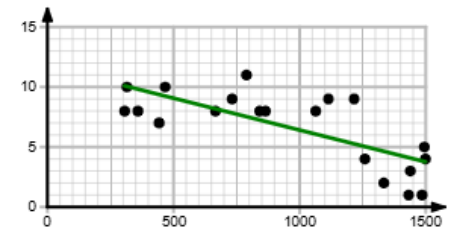


Most points are close to the line of best fit meaning it is **strong positive correlation**.

Question 12

decreases

① There is a downward trend in the data on the scatter diagram. Therefore, as height increases, temperature decreases.

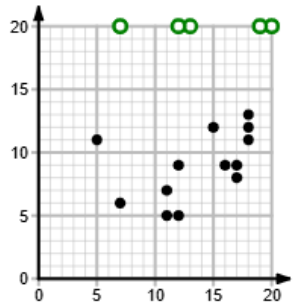


Question 13

27.8%

① The total number of pupils is 18.

② Geography scores are on the y-axis. Identify points with a y-value of 20.



There are 5 pupils with a maximum score in Geography.

③ Calculate the percentage.

$$\frac{\text{count}_{\text{full marks}}}{\text{total}_{\text{students}}} \times 100 = \frac{5}{18} \times 100$$
$$= 27.7778\ldots$$
$$= 27.8\%(\text{to 1 dp})$$

Question 14

n^4

① Write n^5 as n multiplied by itself 5 times and write n as n^1.

$$\frac{n^5}{n^1} = \frac{n \times n \times n \times n \times n}{n}$$

② Cancel out as many n's as possible (or subtract the powers) and write the answer in index form ($n^{\square}$).

$$\frac{n \times n \times n \times n \times n}{n} \equiv n^{5-1}$$
$$n \times n \times n \times n \equiv n^4$$

Question 15

8 or 8^1

① Use laws of indices to simplify.

$$\frac{8^8}{8^4 \times 8^3} = \frac{8^8}{8^7}$$
$$= 8$$

Question 16

$n = 0$

Question 17

$x = 3$

Question 18

1

① Anything raised to the power of 0 is equal to 1

$$\left(7x^5y^5\right)^0 = 1$$

Question 19

$16p^{32}$

① Raise each factor in the bracket to the power of 4

$$\begin{aligned}\left(2p^8\right)^4 &= 2^4\left(p^8\right)^4 \\ &= 2^4p^{(8\times4)} \\ &= 16p^{32}\end{aligned}$$

Question 20

$\frac{1}{5}x^{-7}y^{16}$ or $\frac{y^{16}}{5x^7}$

① Use laws of indices to simplify the denominator.

$$\begin{aligned}&\frac{25x^2y^{16}}{\left(5x^3\right)^3} \\ &= \frac{25x^2y^{16}}{5^3\left(x^3\right)^3} \\ &= \frac{25x^2y^{16}}{125x^{(3\times3)}} \\ &= \frac{25x^2y^{16}}{125x^9}\end{aligned}$$

② Use the laws of indices for division to simplify the fraction.

$$\frac{25x^2y^{16}}{125x^9}$$
$$= \frac{25^{\cancel{25}^{1}}}{\cancel{125}_{5}} x^{(2-9)}y^{16}$$
$$= \frac{1}{5}x^{-7}y^{16}$$

Question 21

$\frac{1}{4x^6}$ or $\frac{1}{4}x^{-6}$

① Take the reciprocal of $\left(2x^3\right)$ due to the negative power.

$$\left(2x^3\right)^{-2} = \frac{1}{\left(2x^3\right)^2}$$

② Raise each term to the power of 2.

$$\frac{1}{\left(2x^3\right)^2} = \frac{1}{2^2(x^3)^2}$$
$$= \frac{1}{4x^6}$$

Question 22

$\frac{t^4w^4}{16}$ or $\frac{1}{16}t^4w^4$

① Simplify $\frac{36t^4w}{9t^6w^3}$

$$\left(\frac{36t^4w}{9t^6w^3}\right)^{-2} = \left(\frac{\cancel{36}^{4}\cancel{t^4}\cancel{w}}{\cancel{9}^{1}\cancel{t^6}\cancel{w^3}^{2}}\right)^{-2}$$
$$= \left(\frac{4}{t^2w^2}\right)^{-2}$$

② Take the reciprocal of $\frac{4}{t^2w^2}$ due to the negative power.

$$\left(\frac{4}{t^2w^2}\right)^{-2} = \left(\frac{t^2w^2}{4}\right)^2$$

③ Raise each part of the term to the power of 2 .

$$\left(\frac{t^2w^2}{4}\right)^2 = \frac{(t^2)^2(w^2)^2}{4^2}$$
$$= \frac{t^4w^4}{16}$$

Question 23

$x = 5$ or $x = -5$

① Use index laws to rewrite x^{-2}

$$\begin{aligned} x^{-2} &= \frac{1}{25} \\ \Rightarrow \frac{1}{x^2} &= \frac{1}{25} \\ \Rightarrow x^2 &= 25 \end{aligned}$$

② Solve to find x.

$$\begin{aligned} \Rightarrow x &= \pm\sqrt{25} \\ \Rightarrow x &= \pm 5 \end{aligned}$$

Question 24

$\frac{3}{8}$

① The power is $\frac{1}{2}$ so take the square root.

$$\begin{aligned} \left(\frac{9}{64}\right)^{\frac{1}{2}} &= \frac{\sqrt{9}}{\sqrt{64}} \\ &= \frac{3}{8} \end{aligned}$$

Question 25

$\frac{1}{16a^{14}b^2}$ or $\frac{1}{16}a^{-14}b^{-2}$

① As the power is negative, take the reciprocal of $64a^{21}b^3$

$$\left(64a^{21}b^3\right)^{-\frac{2}{3}} = \frac{1}{\left(64a^{21}b^3\right)^{\frac{2}{3}}}$$

② Raise each term inside the bracket to the power of $\frac{2}{3}$

$$\begin{aligned} \frac{1}{\left(64a^{21}b^3\right)^{\frac{2}{3}}} &= \frac{1}{64^{\frac{2}{3}}(a^{21})^{\frac{2}{3}}\left(b^3\right)^{\frac{2}{3}}} \\ &= \frac{1}{\left(\sqrt[3]{64}\right)^2 a^{14}b^2} \\ &= \frac{1}{4^2 a^{14}b^2} \\ &= \frac{1}{16a^{14}b^2} \end{aligned}$$

Question 26

$\frac{108}{300}$ or 36%

① Recall the formula to calculate relative frequency.

$$\text{relative frequency} = \frac{\text{number of a specific outcome}}{\text{total number of outcomes}}$$
$$= \frac{\text{number of black spins}}{\text{total number of spins}}$$

② Substitute in the corresponding values.

$$= \frac{108}{300}$$
$$\left(= \frac{9}{25}\right)$$

Question 27

Ciaran

Question 28

$5.25 \leq y < 5.35$

This is as if the number had been rounded to the nearest 0.1.

To find the lower bound you can subtract half the accuracy, 0.05, from the rounded number.

To find the upper bound you add half the accuracy.

Lower bound $= 5.3 - 0.05 = 5.25$

Upper bound $= 5.3 + 0.05 = 5.35$

Therefore the error interval is: $5.25 \leq y < 5.35$

Question 29

Lower bound =4.8
Upper bound =4.9

As the number was truncated to 1 decimal place, all the digits after this accuracy were removed regardless of their value.

Lower bound $= 4.8000000000 \ldots = 4.8$

Upper bound $= 4.8999999999 \ldots = 4.9$

Question 30

16.025

To find the lower bound of p, use the lower bound of q and the lower bound of r.

$$\begin{aligned} p_{lower} &= 5q_{lower}r_{lower} \\ &= 5(3.665)(0.8745) \\ &= 16.02521 \end{aligned}$$

Question 31

1.997

$$\begin{aligned} a_{lower} &= \frac{b_{lower}}{c_{upper} + d_{upper}} \\ &= \frac{(3.5)}{(0.0075) + (1.745)} \\ &= 1.99715 \end{aligned}$$

Question 32

1

$$\begin{aligned} p_{upper} &= \frac{q_{upper}}{r_{lower} + s_{lower}} \\ &= \frac{(0.755)}{(0.55) + (0.0045)} \\ &= 1.13654 \\ p_{lower} &= \frac{q_{lower}}{r_{upper} + s_{upper}} \\ &= \frac{(0.745)}{(0.65) + (0.0055)} \\ &= 1.36159 \end{aligned}$$

(Could not display math) both bounds agree to that level of accuracy.

Question 33

$z = 120^\circ$

① The angle at the centre is twice the angle at the circumference therefore $z = 120^\circ$

Question 34

$x = 61^\circ$

① Calculate the third angle in the triangle.

$$180 - 63 - 56 = 61^\circ$$

② The angle between the tangent and the chord is equal to the angle in the alternate segment.

$$x = 61^\circ$$

Question 35

$x = 120^\circ$
$y = 240^\circ$

① Find x using the fact that opposite angles in a cyclic quadrilateral add to 180°

$$x = 180 - 60$$
$$= 120^\circ$$

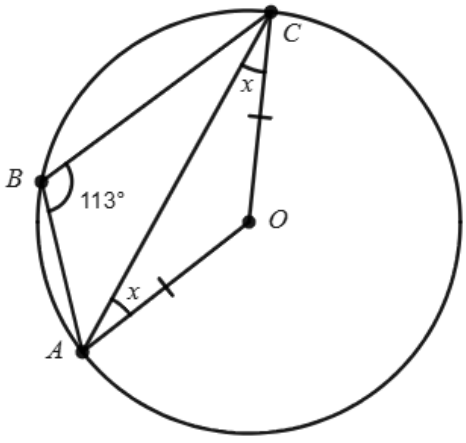
② Find y using the fact that the angle at the centre is double the angle at the circumference.

$$y = 2 \times 120$$
$$= 240^\circ$$

Question 36

$x = 23^\circ$

① OA and OC are both radii of the circle. This means that $\triangle\ OAC$ is an isosceles triangle.



② The angle at the centre is twice the angle at the circumference therefore

$$\angle AOC = 2 \times 113$$
$$\angle AOC = 226$$

③ The angle $\angle AOC$ inside the $\triangle\ OAC$ is given by

$$360\ -\ \angle AOC\ =\ 134$$

④ The angles in a triangle sum to 180 and the base angles of an isosceles triangle are equal.

$$\begin{aligned} 2\,x\ +\ 134\ &=\ 180 \\ x\ &=\ 23 \end{aligned}$$

Question 37

x =60 °

① Find $\angle ACB$ using the fact that angles in the same segment are equal.

$$\angle ACB\ =\ 30^\circ$$

② Find $\angle ACD$ using the fact that angle in a semicircle is 90°.

$$\angle ACD\ =\ 90^\circ$$

③ Find x.

$$\begin{aligned} x\ &=\ 90\ -\ 30 \\ &=\ 60^\circ \end{aligned}$$

Question 38

$\angle BCA$ =58 °

① Find $\angle CBA$ using the fact that opposite angles in a cyclic quadrilateral add up to 180°.

$$\begin{aligned} \angle CBA\ &=\ 180\ -\ 113^\circ \\ &=\ 67^\circ \end{aligned}$$

② Find $\angle BAC$ using the alternate segment theorem.

$$\angle BAC\ =\ 55^\circ$$

③ Find $\angle BCA$ using the fact that angles in a triangle add up to 180°.

$$\begin{aligned} \angle BCA\ &=\ 180\ -\ 55\ -\ 67 \\ &=\ 58^\circ \end{aligned}$$

Question 39

c =85 °

Question 40

$V_0 = 240000$
$V_{n+1} = 1.02V_n$, where $n \geq 0$

Question 41

$b = -9$

Question 42

$a = 7$
$b = 1$
$c = 2$

Question 43

When $x = 0$, $y = -1$
When $x = 1$, $y = 1$
As there is a change in sign, there must be at least one root between $x = 0$ and $x = 1$.

Question 44

$x_3 = -2.008$

① Input 2 in your calculator and press [=] or [EXE].

② Input the following on your calculator.

$$\sqrt[3]{\frac{-5(\text{Ans})^2 - 4\text{Ans} - 1}{2}}$$

③ Press [=] or [EXE] three times.

$$x_1 = -2.438499$$
$$x_2 = -2.188974$$
$$x_3 = -2.008387$$

Question 45

£1643.34

① Calculate the decimal multiplier equivalent to an increase of $4\,\%$

$$\frac{100+4}{100} = 1.04$$

② Use the formula for compound interest to calculate the final amount.

$$780 \times 1.04^{19} = 1643.3423 \ldots$$
$$\approx pound\ 1643.34$$

Question 46

£1048.47

① Apply the decimal multipliers for each percentage change.

$$1225 \times 0.99^5 \times 0.9^1$$
$$= 1048.47$$

Question 47

Account A

Question 48

$r = 3.14$

Question 49

12

① Multiply the number of cards in each bag together.

$$3 \times 4 = 12$$

Question 50

12decals

① Form and solve an equation to describe the scenario.

$$\begin{aligned} \text{colour} \times \text{wheels} \times \text{decals} &= 1152 \\ 12 \times 8 \times d &= 1152 \\ 96d &= 1152 \\ \text{decals} &= 12 \end{aligned}$$

Question 51

36choices

① Work out and add up all the possible combinations.

$$\begin{aligned} \text{Starters} \times \text{Mains} &= 3 \times 6 \\ &= 18 \\ \text{Mains} \times \text{Desserts} &= 6 \times 3 \\ &= 18 \\ \\ \text{Total} &= 18 + 18 \\ &= 36 \end{aligned}$$

Question 52

99choices

① Work out and add up all the possible combinations.

$$\begin{aligned} \text{cases} \times \text{protectors} &= 9 \times 3 \\ &= 27 \\ \text{cases} \times \text{gifts} &= 9 \times 6 \\ &= 54 \\ \text{protectors} \times \text{gifts} &= 3 \times 6 \\ &= 18 \\ \\ \text{Total} &= 27 + 54 + 18 \\ &= 99 \end{aligned}$$

Question 53

2052lists

① Consider the combinations for boy, girl, boy.

$$\begin{aligned} &\text{Boy} \times \text{Girl} \times \text{Boy} \\ &= 9 \times 12 \times 8 \\ &= 864 \end{aligned}$$

② Consider the combinations for girl, boy, girl.

$$\begin{aligned} &\text{Girl} \times \text{Boy} \times \text{Girl} \\ &= 12 \times 9 \times 11 \\ &= 1188 \end{aligned}$$

③ Add the two answers together.

$$\begin{aligned} &864 + 1188 \\ &= 2052 \end{aligned}$$

Question 54

$\frac{10}{24}$

① Work out the denominator by finding the total amount of 2 digit numbers which can be made.

For the first number there are 2 choices.

For the second number there are now 1 choices and so on.

$$\begin{aligned}&\text{Different 4 digit numbers}\\&= 4 \times 3 \times 2 \times 1\\&= 24\end{aligned}$$

② To work out the numerator, first consider the case where the first digit is odd.

For the first digit, there is 1 odd number less than 2.
For the last digit, there are now 2 odd numbers to choose from.

$$\begin{aligned}&\text{Choices for first number} = 1\\&\text{Choices for last number} = 2\end{aligned}$$

For the remaining 2 numbers there are 2 and 1 choices respectively.

Choices for each digit:

$$\underline{1} \times \underline{2} \times \underline{1} \times \underline{2}$$

$$= 4 \text{ choices}$$

③ Consider the case where the first digit is even.

For the first digit, there is 1 even number less than 7.
For the last digit, there are 3 odd numbers to choose from.

$$\begin{aligned}&\text{Choices for first number} = 1\\&\text{Choices for last number} = 3\end{aligned}$$

For the remaining 2 numbers there are 2 and 1 choices respectively.

Choices for each digit:

$$\underline{1} \times \underline{2} \times \underline{1} \times \underline{3}$$

$$= 6 \text{ choices}$$

④ Add the two answers together.

$$\begin{aligned}&4 + 6\\&= 10\end{aligned}$$

⑤ Write the answer as a fraction.

$$\frac{10}{24} \left(= \frac{5}{12} \right)$$

Question 55

120cm ^{2}

① Find the length scale factor from triangle A to triangle B.

$$\begin{aligned}\text{length}_{\text{SF}} &= 14 \div 8.4 \\ &= \frac{5}{3}\end{aligned}$$

② Find the area scale factor from triangle A to triangle B.

$$\begin{aligned}\text{area}_{\text{SF}} &= \left(\frac{5}{3}\right)^2 \\ &= \frac{25}{9}\end{aligned}$$

③ Use the area scale factor to find the area of triangle B.

$$\begin{aligned}\text{area}_B &= 43.2 \times \frac{25}{9} \\ &= 120 \text{ cm}^2\end{aligned}$$

Question 56

9cm

① Use the corresponding volumes to find the volume scale factor from shape A to shape B.

$$\begin{aligned}\text{volume sf} &= 1166.2 \div 734.4 \\ &= \frac{343}{216}\end{aligned}$$

② Cube root the volume scale factor to find the length scale factor.

$$\begin{aligned}\text{length sf} &= \sqrt[3]{\frac{343}{216}} \\ &= \frac{7}{6}\end{aligned}$$

③ Divide the height of B by the length scale factor to find the height of A.

$$\begin{aligned}\text{height}_A &= 10.5 \div \frac{7}{6} \\ &= 9 \text{ cm}\end{aligned}$$

Question 57

142.1cm ^{2}

① Use the corresponding volumes to find the volume scale factor from shape A to shape B.

$$\begin{aligned}\text{volume sf} &= 27 \div 343 \\ &= \frac{27}{343}\end{aligned}$$

② Cube root the volume scale factor to find the length scale factor, and then square this to find the area scale factor.

$$\begin{aligned}\text{length sf} &= \sqrt[3]{\frac{27}{343}} \\ &= \frac{3}{7} \\ \text{area sf} &= \left(\frac{3}{7}\right)^2 \\ &= \frac{9}{49}\end{aligned}$$

③ Divide the surface area of B by the area scale factor to find the surface area of A.

$$\begin{aligned}\text{surface area}_A &= 26.1 \div \frac{9}{49} \\ &= 142.1 \text{ cm}^2\end{aligned}$$

Question 58

$1:9$

① Find the surface area scale factor from A to B by squaring the length scale factor.

$$\begin{aligned}\text{area}_{\text{SF}} &= 3^2 \\ &= 9\end{aligned}$$

② Use the surface area scale factor to find the ratio of the surface area of A to the surface area of B.

$$\begin{aligned}&\text{surface area of } A : \text{surface area of } B \\ &1 : 9\end{aligned}$$

Question 59

156.8cm ^{2}

① Use BC and XY to find the length scale factor from trapezium $OABC$ to trapezium $OWXY$.

$$\begin{aligned}\text{length}_{\text{SF}} &= XY \div BC \\ &= 23 \div 9 \\ &= \frac{23}{9}\end{aligned}$$

② Square the length scale factor to find the area scale factor from trapezium $OABC$ to trapezium $OWXY$.

$$\text{area}_{\text{SF}} = \left(\frac{23}{9}\right)^2$$
$$= \frac{529}{81}$$

③ Use the area scale factor to find the area of trapezium $OABC$.

$$\text{area}_{OABC} = \text{area}_{OWXY} \div \frac{529}{81}$$
$$= 185.15 \div \frac{529}{81}$$
$$= 28.35 \text{ cm}^2$$

④ Subtract the area of trapezium $OABC$ from the area of trapezium $OWXY$ to find the shaded area.

$$\text{shaded area} = 185.15 - 28.35$$
$$= 156.8 \text{ cm}^2$$

Question 60

46.7cm ^{3}

① Use the diameters of the two cones to find the length scale factor from the smaller to larger cone.

$$\text{length}_{\text{SF}} = 5.6 \div 3.2$$
$$= \frac{7}{4}$$

② Call the height of the small cone h. Use the length scale factor to form an equation in h.

$$h_{\text{large}} = h_{\text{small}} \times \frac{7}{4}$$
$$h + 3 = \frac{7}{4}h$$
$$-h \downarrow \quad \downarrow -h$$
$$3 = \frac{3}{4}h$$
$$\div\frac{3}{4} \downarrow \quad \downarrow \div\frac{3}{4}$$
$$4 = h$$
$$h_{\text{small}} = 4 \text{ cm}$$
$$h_{\text{large}} = 4 + 3 = 7 \text{ cm}$$

③ Find the radius of each cone.

$$r_{\text{large}} = 5.6 \div 2 = 2.8 \text{ cm}$$
$$r_{\text{small}} = 3.2 \div 2 = 1.6 \text{ cm}$$

④ Use the formula $V = \frac{1}{3}\pi r^2 h$ to find the volume of each cone.

$$V_{\text{large}} = \frac{1}{3}\pi \times 2.8^2 \times 7$$
$$= 57.4702\ldots$$
$$V_{\text{small}} = \frac{1}{3}\pi \times 1.6^2 \times 4$$
$$= 10.7233\ldots$$

⑤ Find the volume of the frustum.

$$V_{\text{frustum}} = V_{\text{large}} - V_{\text{small}}$$
$$= 57.4702\ \ldots - 10.7233\ \ldots$$
$$= 46.7469\ldots$$
$$= 46.7\ \text{cm}^3$$

Question 61

7.87×10^6

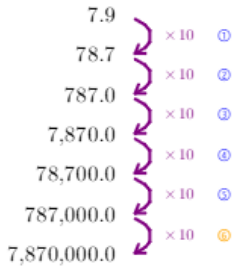
① Identify the digit that will be in the ones place value for standard form.

$\underline{7}\,870000$

② Use this information to write down the structure for the standard form.

$7870000 = \underline{7}.87 \times 10^{\square}$

③ Count the number of multiplications by 10 needed to reach 7870000 from 7.87



④ Write the answer.

$7870000 = \underline{7}.87 \times 10^6$

Question 62

4.61×10^{-4}

① Convert 0.00461 to standard form.

$0.00461 = \underline{4.61 \times 10^{-3}}$

② Replace 0.00461 in the question with its standard form.

$$0.00461 \times 10^{-1} = \underline{4.61 \times 10^{-3}} \times 10^{-1}$$

③ Use index laws with the powers of 10.

$$10^{-3} \times 10^{-1} = 10^{-4}$$

④ Write the answer.

$$0.00461 \times 10^{-1} = 4.61 \times 10^{-4}$$

Question 63

4×10^{-22}

① Multiply the coefficients.

$$8 \times 5 = 40$$

② Write the result of the multiplication in standard form.

$$40 = 4 \times 10^{1}$$

③ Use index laws with all the powers of 10.

$$10^{-4} \times 10^{-19} \times 10^{1} = 10^{-22}$$

④ Write the answer.

$$\text{Answer} \rightarrow 4 \times 10^{-22}$$

Question 64

539hours

① Calculate the scale factor by dividing the final length of wall by the initial length of wall then simplifying the resulting fraction.

$$\text{scale factor} = 77 \div 70$$
$$= \frac{11}{10}$$

② A scale factor of $\frac{11}{10}$ is equivalent to multiplying by 11 and dividing by 10.

③ Write a ratio table and use the scale factor to find the required time.

70m	$\overset{\div 10}{\rightarrow}$	7m	$\overset{\times 11}{\rightarrow}$	77m

490hours	$\overset{\div 10}{\rightarrow}$	49hours	$\overset{\times 11}{\rightarrow}$	539hours

Question 65

850g flour
700ml cream
900g sugar
900g chocolate
500g fruits

① Find the scale factor.

$$5 \text{ people} \overset{\times 5}{\rightarrow} 25 \text{ people}$$

You need 5 times the amount of each ingredient.

② Multiply each quantity by 5 .

$170 \times 5 = 850$ g flour
$140 \times 5 = 700$ ml cream
$180 \times 5 = 900$ g sugar
$180 \times 5 = 900$ g chocolate
$100 \times 5 = 500$ g fruits

Question 66

6classrooms

① Work out how many classrooms would be needed if the class size was 1.

$$\begin{array}{cc} 18 \text{ pupils} & 5 \text{ classrooms} \\ \div 18 \downarrow & \downarrow \times 18 \\ 1 \text{ pupil} & 90 \text{ classrooms} \end{array}$$

② Now work out how classrooms would be needed if the class size was 15 .

$$\begin{array}{cc} 1 \text{ pupil} & 90 \text{ classrooms} \\ \times 15 \downarrow & \downarrow \div 15 \\ 15 \text{ pupils} & 6 \text{ classrooms} \end{array}$$

Question 67

Inverse

Question 68

$y =$12.2cm

① Use Pythagoras' theorem.

$$\begin{aligned} a^2+b^2 &= c^2 \\ 7^2+10^2 &= y^2 \\ 149 &= y^2 \\ y &= \sqrt{149} \\ &= 12.207\ldots \\ &= 12.2\ \text{cm} \end{aligned}$$

Question 69

11.0cm

① Use Pythagoras' theorem.

$$\begin{aligned} a^2+b^2 &= c^2 \\ y^2+7^2 &= 13^2 \\ y^2 &= 13^2-7^2 \\ y^2 &= 120 \\ y &= \sqrt{120} \\ &= 10.954\ldots \\ &\approx 11.0\ \text{cm} \end{aligned}$$

Question 70

The triangle can be constructed.

① Find the values of AB^2, BC^2 and AC^2.

$$\begin{aligned} AB^2 &= 20^2 \\ &= 400 \\ BC^2 &= 37.5^2 \\ &= 1406.25 \\ AC^2 &= 42.5^2 \\ &= 1806.25 \end{aligned}$$

② Compare the value of AB^2+BC^2 with the value of AC^2.

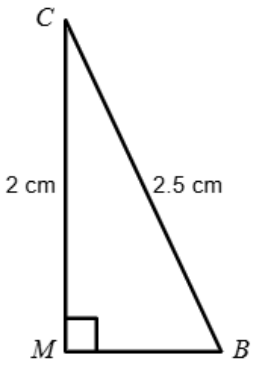
$$\begin{aligned} AB^2+BC^2 &= 400+1406.25 \\ &= 1806.25 \\ &= AC^2 \end{aligned}$$

Therefore, by the converse of Pythagoras' theorem, the triangle with the given lengths and angle can be constructed.

Question 71

3cm ^{2}

① Use the line of symmetry of the isosceles triangle to draw a right-angled triangle, with the right angle at the midpoint of AB.



② Use Pythagoras' Theorem to find the missing side in this triangle.

$$\begin{aligned} MB\ 2 &= 2.5\ 2 - 2\ ^2 \\ &= 2.25 \\ M\,B\ &= 1.5\ cm \end{aligned}$$

③ Find the area of triangle ABC.

$$\begin{aligned} Area\ &= \tfrac{1}{2} \times 3 \times 2 \\ &= 3\ cm\ ^2 \end{aligned}$$

Question 72

£240

① Use Pythagoras' theorem to find the length XZ.

$$\begin{aligned} X\,Z\ 2\ &= 3.4\ ^2 + 5.5^2 \\ &= 41.81 \\ X\,Z\ &= 6.4661\ \ldots \end{aligned}$$

② Find the total length of all the rods in the framework.

$$\begin{aligned} total\ \ length\ &= XY + YZ + XZ \\ &= 3.4 + 5.5 + 6.4661\ \ldots \\ &= 15.3661\ \ldots \end{aligned}$$

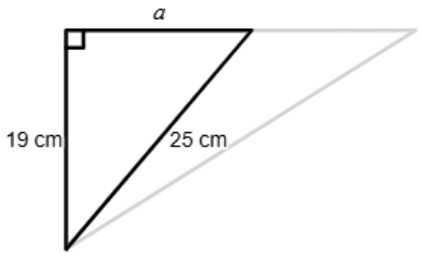
③ Round this value **up** to the nearest metre and multiply by the cost per metre.

$$\begin{aligned} total\ \ cost &= 16 \times 15 \\ &= £\,240 \end{aligned}$$

Question 73

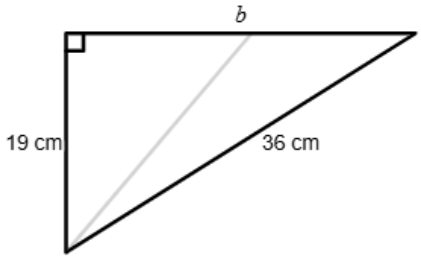
y =14.3cm

① Use Pythagoras' theorem to find a.



$$\begin{aligned} a^2 &= 25^2 - 19^2 \\ a &= \sqrt{264} \\ &= 16.2481 \end{aligned}$$

② Use Pythagoras' theorem to find b.



$$\begin{aligned} b^2 &= 36^2 - 19^2 \\ b &= \sqrt{935} \\ &= 30.5778 \end{aligned}$$

③ Subtract to find y.

$$\begin{aligned} y &= 30.5778 - 16.2481 \\ &= 14.3\ \text{cm} \end{aligned}$$

Question 74

47cm

① Use Pythagoras' theorem in triangle PQR to find the radius of the quarter circle.

$$\begin{aligned} QR\ 2 &= 12.5^{\,2} - 4.5^{\,2} \\ &= 136 \\ QR &= 11.6619\ \ldots \end{aligned}$$

② Find the circumference of the circle and divide it by four to find the length of the arc SR.

$$\begin{aligned} C &= 2\pi r \\ &= 2 \times \pi \times 11.6619 \ldots \\ &= 73.2739 \ldots \\ &\downarrow \div 4 \\ arc\ RS &= 18.3185 \ldots \end{aligned}$$

③ Add PQ, QS (which is the radius of the circle), the arc SR and RP to find the perimeter of the shape.

$$\begin{aligned} P &= 4.5 + 11.6619 \ldots + 18.3185 \ldots + 12.5 \\ &= 46.9804 \ldots \end{aligned}$$

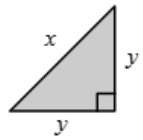
④ Round the answer to 1 decimal place.

$$P = 47\ cm$$

Question 75

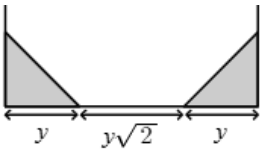
1.46cm

① Use Pythagoras' theorem to express the side length, x, of the octagon in terms of y.



$$\begin{aligned} x^2 &= y\ 2 + y\ 2 \\ &= 2y^2 \\ x &= y\sqrt{2} \end{aligned}$$

② Express the side length of the square in terms of y and equate to 5.



$$\begin{aligned} y + y\sqrt{2} + y &= 5 \\ (2 + \sqrt{2})y &= 5 \\ y &= \frac{5}{2+\sqrt{2}} \\ &= 1.4645 \ldots \\ y &= 1.46\ \text{cm} \end{aligned}$$

Question 76

13.5cm

① Find the diagonal, AG, of the cuboid using $\sqrt{a^2+b^2+c^2}$, where a is the length, b is the height and c is the depth of the cuboid.

$$\begin{aligned} AG &= \sqrt{6^2+8^2+9^2} \\ &= \sqrt{181} \\ &= 13.4536 \\ &= 13.5 \text{ cm} \end{aligned}$$

Question 77

12.2cm

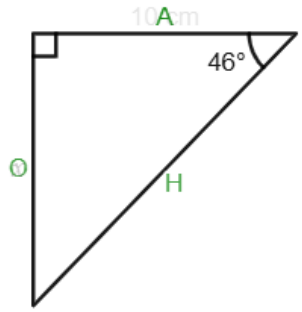
Question 78

10.9cm

Question 79

x =10.4cm

① Label the sides.



② Decide the trigonometric ratio to use.

SOH CAH T**OA**
Therefore we use tan

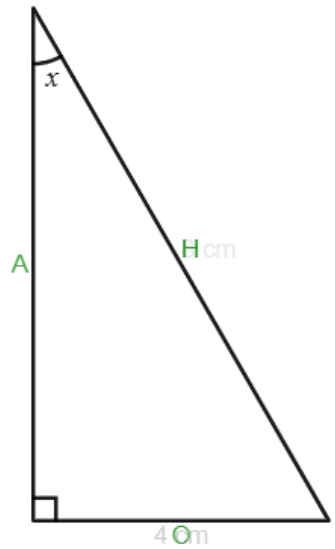
③ Write an equation and solve.

$$\begin{aligned} \tan(\theta) &= \frac{O}{A} \\ \tan(46) &= \frac{x}{10} \\ x &= 10\tan(46) \\ &= 10.4 \text{ cm} \end{aligned}$$

Question 80

x =30°

① Label the sides.



② Decide the trigonometric ratio to use.

S**OH** CAH TOA
Therefore we use sin

③ Write an equation and solve.

$$\begin{aligned}\sin(\theta) &= \frac{O}{H}\\ \sin(x) &= \frac{4}{8}\\ x &= \sin^{-1}\left(\frac{4}{8}\right)\\ &= 30^{\circ}\end{aligned}$$

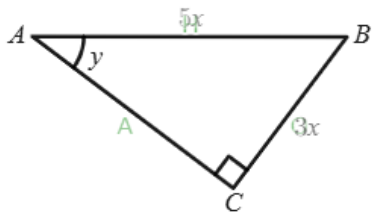
Question 81

$x = 4.2$ cm

Question 82

$y = 36.9^{\circ}$

① Label the sides.



② Decide the trigonometric ratio to use.

SOH CAH TOA
Therefore we use sin

③ Write an equation and solve.

$$\begin{aligned}\sin(\theta) &= \frac{O}{H}\\ \sin(y) &= \frac{3x}{5x}\\ y &= \sin^{-1}\left(\frac{3}{5}\right)\\ &= 36.9^{\circ}\end{aligned}$$

Question 83

44.0cm

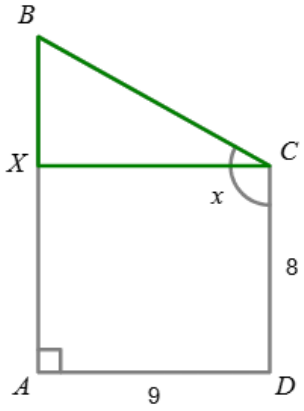
Question 84

$x = 119.1^{\circ}$

① Use the area to find the height of the trapezium.

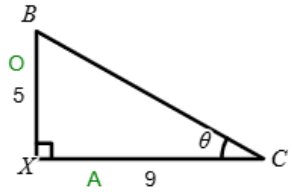
$$\begin{aligned}Area &= \frac{a+b}{2} \times h\\ 94.5 &= \frac{13+8}{2} \times h\\ h &= \frac{94.5 \times 2}{13+8}\\ h &= 9\end{aligned}$$

② Create a triangle BCX and find the length of BX.



$$\begin{aligned} AB &= AX + BX \\ &= CD + BX \\ 13 &= 8 + BX \\ BX &= 5 \end{aligned}$$

③ Find the angle at C of the triangle BCX.



$$\begin{aligned} \tan(\theta) &= \frac{O}{A} \\ \tan(\theta) &= \frac{5}{9} \\ \theta &= 29.1\ldots \end{aligned}$$

④ Find the value of x

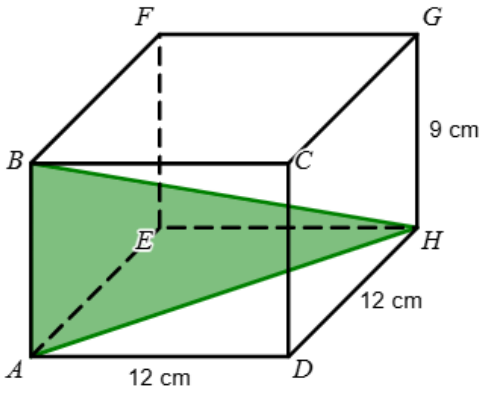
$$\begin{aligned} x &= 90 + 29.1\ldots \\ x &= 119.1^{\circ} \end{aligned}$$

Question 85

32.9km

Question 86

27.9°



① Find the length of AH using Pythagoras' theorem in triangle ADH:

$$\begin{aligned} AH &= \sqrt{12^{\,2} + 12'^{2}} \\ &= \sqrt{288} \\ &= 16.9706\ldots \end{aligned}$$

② Find $\angle BHA$ using trigonometry in triangle ABH:

$$\begin{aligned} \tan(\angle BHA) &= \frac{9}{16.9706\ldots} \\ \angle BHA &= \tan^{-1}\left(\frac{9}{16.9706\ldots}\right) \\ &= 27.9^{\circ} \text{ (to 1 dp)} \end{aligned}$$

';

Question 87

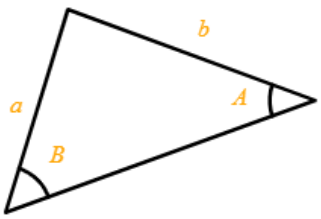
61.9cm

Question 88

z =8.0cm

① Recall the formula for the sine rule to find a missing length and label the triangle accordingly.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$



② Substitute the values into the sine rule.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\Rightarrow \frac{z}{\sin(40)} = \frac{10}{\sin(53)}$$

③ Solve for z.

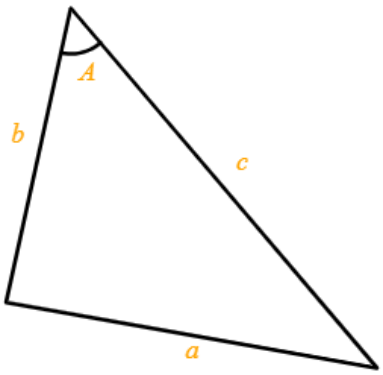
$$\frac{z}{\sin(40)} = \frac{10}{\sin(53)}$$
$$\times \sin(40) \downarrow \quad \downarrow \times \sin(40)$$
$$z = \frac{10}{\sin(53)} \times \sin(40)$$
$$\approx 8.0$$

Question 89

z =8.8cm

① Recall the formula for the cosine rule and label the triangle accordingly.

$$a^2 = b^2 + c^2 - 2bc\cos A$$



② Substitute the values into the cosine rule.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Rightarrow z^2 = 7^2 + 11^2 - 2 \times 7 \times 11 \times \cos(53)$$

③ Calculate and square root.

$$z = \sqrt{7^2 + 11^2 - 2 \times 7 \times 11 \times \cos(53)}$$
$$= \sqrt{77.32\ldots}$$
$$\approx 8.8 \text{ cm}$$

Question 90

y =107.1 °

Taking y'as an acute angle, using the sine rule, $\frac{\sin y'}{24.2} = \frac{\sin(26)}{11.1}$

Multiply both sides by 24.2

$$\sin y' = \frac{24.2 \times \sin(26)}{11.1}$$
$$y' = \sin^{-1}\left(\frac{24.2 \times \sin(26)}{11.1}\right)$$
$$y' = 72.9^\circ$$
$$y = 180 - 72.9 = 107.1^\circ$$

Question 91

73.4°

Question 92

42.3cm^2

$$\text{Area} = \frac{1}{2} \times 8 \times 11 \times \sin 74$$
$$= 42.3 \text{ cm}^2$$

Question 93

x =6.0cm

① Use the sine rule to find the angle opposite to 5 cm.

$$\frac{\sin p}{5} = \frac{\sin 93}{8}$$
$$p = \sin^{-1}\left(\frac{5 \times \sin 93}{8}\right)$$
$$= 38.619\ldots^\circ$$

② Find the angle opposite to x.

$$q = 180 - 93 - 38.619\ldots$$
$$= 48.381\ldots^\circ$$

③ Use the cosine rule to find x.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$x^2 = 8^2 + 5^2 - 2 \times 8 \times 5\cos 48.381\ldots.$$
$$x = 5.989\ldots \text{ cm}$$

Question 94

Area = 374cm^2

Question 95

22.9cm ^{2}

① Calculate the radius.

$$\frac{101}{360} \times \pi r^2 - \frac{1}{2} \times r^2 \sin(101) = 66$$
$$r^2 \left(\frac{101}{360} \pi - \frac{1}{2} \sin(101) \right) = 66$$
$$r^2 = \frac{66}{\left(\frac{101}{360} \pi - \frac{1}{2} \sin(101) \right)}$$
$$r = 12.9992$$

② Find the length of the arc.

$$l = \frac{101}{360} \times 2\pi \times 12.9992$$
$$= 22.9 \text{ cm}$$

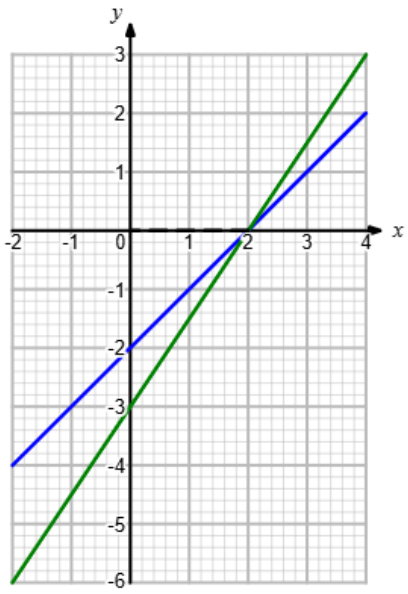
Question 96

2

Question 97

x =2
y =0

① Find the coordinates of the point of intersection.



$x = 2, y = 0$

Question 98

$x = 5, y = -4$

① Label the equations.

$$\begin{cases} 3x + 4y = -1 \ ① \\ 5x + 4y = 9 \ ② \end{cases}$$

② Evaluate ② − ① to eliminate y and then solve for x.

$$\begin{aligned} ② - ① \Rightarrow \ 2x &= 10 \\ x &= 5 \end{aligned}$$

③ Substitute $x = 5$ into ① and solve for y.

$$\begin{aligned} ① \Rightarrow \ 3(5) + 4y &= -1 \\ 15 + 4y &= -1 \\ 4y &= -16 \\ y &= -4 \end{aligned}$$

Question 99

$p = 1, q = -2$

① Label the equations.

$$\begin{cases} 3p + q = 1 \ ① \\ -5p + 2q = -9 \ ② \end{cases}$$

② Multiply equation ① by 2 to match the q coefficients.

$$2 \times ① \Rightarrow \ 6p + 2q = 2 \ ③$$

③ Evaluate ③ − ② to eliminate q and find p.

(Could not display math)

$$\begin{aligned} ③ - ② \Rightarrow \ 11p &= 11 \\ p &= 1 \end{aligned}$$

④ Substitute $p = 1$ into ① and solve for q.

$$\begin{aligned} ① \Rightarrow \ 3(1) + q &= 1 \\ 3 + q &= 1 \\ q &= -2 \end{aligned}$$

Question 100

$x = 1, y = -2$

① Rearrange both equations into the form $ax + by = c$

$$-3x + 2y + 7 = 0$$
$$\Rightarrow -3x + 2y = -7 \quad ①$$

$$2x + 3y + 4 = 0$$
$$\Rightarrow 2x + 3y = -4 \quad ②$$

② Multiply equation ① by 3 and equation ② by 2 to match the y coefficients.

$$3 \times ① \Rightarrow -9x + 6y = -21 \quad ③$$
$$2 \times ② \Rightarrow 4x + 6y = -8 \quad ④$$

③ Evaluate ④ − ③ to eliminate y and find x.

(Could not display math)

$$④ - ③ \Rightarrow 13x = 13$$
$$x = 1$$

④ Substitute $x = 1$ into ① and solve for y.

$$① \Rightarrow -3(1) + 2y = -7$$
$$-3 + 2y = -7$$
$$2y = -4$$
$$y = -2$$

Question 101

$x = 14, y = 26$

① Use the fact the co-interior (allied) angles sum to 180^o to form simultaneous equations.

$$(6x - 18) + (6y - 42) = 180$$
$$6x + 6y - 60 = 180$$
$$6x + 6y = 240 \quad ①$$

$$(x + 18) + (5y + 18) = 180$$
$$x + 5y + 36 = 180$$
$$x + 5y = 144 \quad ②$$

③ Multiply equation ② by 6 to match the x coefficients.

$$6 \times ② \Rightarrow 6x + 30y = 864 \quad ③$$

④ Evaluate ③ − ① to eliminate x and find y.

(Could not display math)

$$③ - ① \Rightarrow \quad 24y = 624$$
$$y = 26$$

⑤ Substitute $y = 26$ into ① and solve for x.

$$① \Rightarrow \quad 6x + 6(26) = 240$$
$$6x + 156 = 240$$
$$6x = 84$$
$$x = 14$$

Question 102

$x = \frac{1}{2}, y = -\frac{7}{2}$ or $x = 6, y = 2$

Equate the right-handside of the two equations and solve the resulting quadratic equation.

$$2x^2 - 12x + 2 = x - 4$$
$$2x^2 - 13x + 6 = 0$$
$$(2x - 1)(x - 6) = 0$$
$$x = \frac{1}{2} \text{ or } x = 6$$

Substitute each value of x into $y = x - 4$

When $x = \frac{1}{2}$, $y = \frac{1}{2} - 4 = -\frac{7}{2}$

When $x = 6$, $y = 6 - 4 = 2$

Question 103

$x = 0, y = 3$ or $x = -3, y = 0$

Substitute $y = x + 3$ into $x^2 + y^2 = 9$ then solve.

$$x^2 + (x + 3)^2 = 9$$
$$x^2 + (x + 3)(x + 3) = 9$$
$$x^2 + x^2 + 3x + 3x + 9 = 9$$
$$2x^2 + 6x + 9 = 9$$
$$2x^2 + 6x = 0$$
$$x^2 + 3x = 0$$
$$x(x + 3) = 0$$
$$\therefore \ x = 0 \text{ or } x = -3$$

Substitute these values into $y = x + 3$

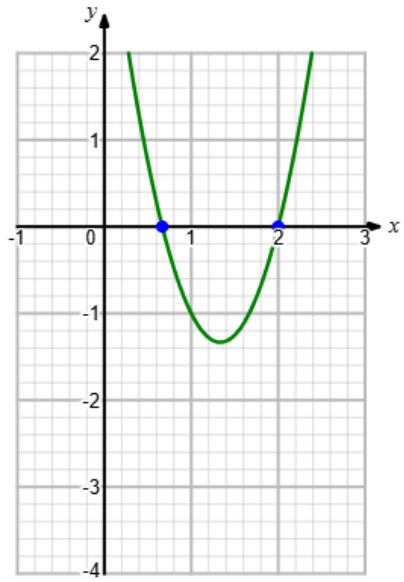
When $x = 0,\ y = 1(0) + 3$
$= 3$

When $x = -3,\ y = 1(-3) + 3$
$= 0$

Question 104

$x \approx$0.5 to 0.9
$x \approx$1.8 to 2.2

① Find the x coordinates where the curve intercepts the x-axis.

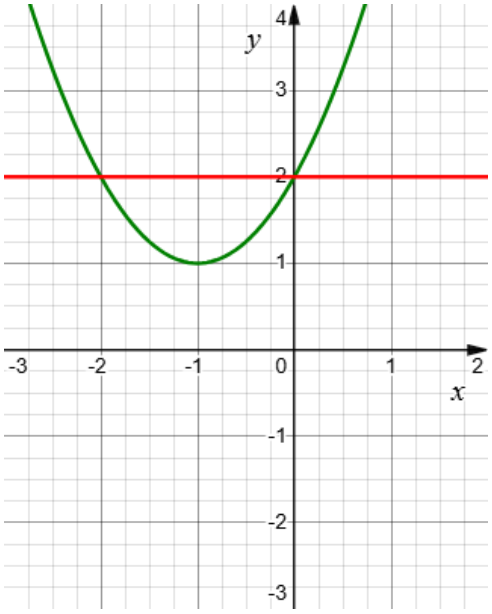


$x \approx 0.7$
$x \approx 2$

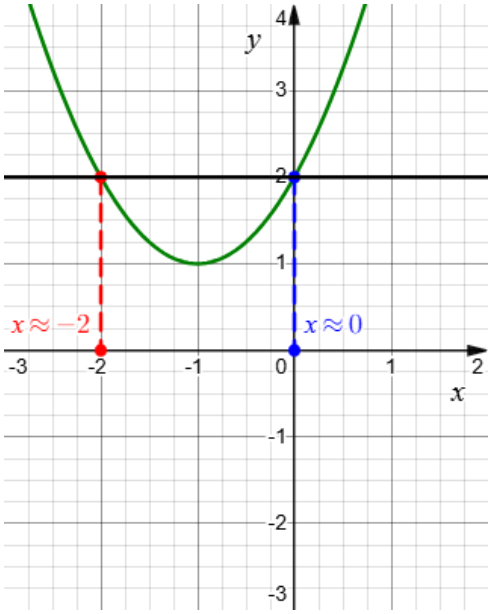
Question 105

$x \approx$-0.2 to 0.2
$x \approx$-2.2 to -1.8

① Plot the horizontal line $y = 2$



② Find the x coordinates where the curve intercepts the horizontal line $y = 2$



$x \approx -2$
$x \approx 0$

Question 106

$\frac{3(x+3)}{4}$

Replace $g(x)$ by y and make x the subject.

$$\begin{aligned} g(x) &= \frac{4x}{3} - 3 \\ y &= \frac{4x}{3} - 3 \\ y + 3 &= \frac{4x}{3} \\ 3(y+3) &= 4x \\ \frac{3(y+3)}{4} &= x \end{aligned}$$

Interchange x and y and then replace y by $g^{-1}(x)$

$$\begin{aligned} \frac{3(x+3)}{4} &= y \\ g^{-1}(x) &= \frac{3(x+3)}{4} \end{aligned}$$

Question 107

$12x^2 + 16x\,y$

① Multiply all terms in the bracket by $4x$, and then simplify.

$$\begin{aligned} 4x\,(3x + 4y) &= 4x \times 3x + 4x \times 4y \\ &= 12x^2 + 16x\,y \end{aligned}$$

Question 108

$21a - 7t + 11$

① Expand the bracket.

$$\begin{aligned} &7(3a - t + 1) + 4 \\ &= 7 \times 3a + 7 \times -t + 7 \times 1 + 4 \\ &= 21a - 7t + 7 + 4 \end{aligned}$$

② Collect like terms and simplify.

$$\begin{aligned} &21a - 7t + 7 + 4 \\ &= 21a - 7t + 11 \end{aligned}$$

Question 109

$22m - 10$

① Expand both brackets.

$$\begin{aligned} &5(5m - 3) - (3m - 5) \\ &= 5(5m - 3) - 1(3m - 5) \\ &= 25m - 15 - 3m + 5 \end{aligned}$$

② Collect like terms.

$$25m-15-3m+5$$
$$= 22m-10$$

Question 110

$x^2 + 6x - 27$

Question 111

$x^2 + 4x + 4$

① Multiply each term in the first bracket by each term in the second bracket.

$$(x+2)^2$$
$$= (x+2)(x+2)$$
$$= x \times x + x \times 2 + 2 \times x + 2 \times 2$$
$$= x^2 + 2x + 2x + 4$$

② Simplify.

$$= x^2 + 4x + 4$$

Question 112

$15x^3 + 50x^2 - 125x$

① Expand $5x(3x-5)$

$$5x\,(3x-5)\;\;(x+5)$$
$$= \;(15x^2 - 25x)\;\;(x+5)$$

② Expand and simplify the remaining brackets.

$$(15x^2 - 25x)\;\;(x+5)$$
$$= 15x^2 \times x + 15x^2 \times 5 - 25x \times x - 25x \times 5$$
$$= 15x^3 + 75x^2 - 25x^2 - 125x$$
$$= 15x^3 + 50x^2 - 125x$$

Question 113

$y^3 + y^2 - 9y - 9$

① Expand the first two brackets.

$$(y+3)(y-3)(y+1)$$
$$= (y^2 - 3y + 3y - 9)\;(y+1)$$
$$= (y^2 - 9)\;(y+1)$$

② Expand the remaining two brackets.

$$(y^2 - 9)(y + 1)$$
$$= y^3 + y^2 - 9y - 9$$

Question 114

$$64x^3 + 144x^2 + 108x + 27$$

① Write as the product of three brackets and expand the first two.

$$(4x + 3)^3$$
$$= (4x + 3)(4x + 3)(4x + 3)$$
$$= \left(16x^2 + 12x + 12x + 9\right)(4x + 3)$$
$$= \left(16x^2 + 24x + 9\right)(4x + 3)$$

② Expand the remaining two brackets.

$$\left(16x^2 + 24x + 9\right)(4x + 3)$$
$$= 64x^3 + 48x^2 + 96x^2 + 72x + 36x + 27$$
$$= 64x^3 + 144x^2 + 108x + 27$$

Question 115

$$4t(3t - 8)$$

Question 116

$$4(4p - 7)$$

Question 117

$$(x - 6)(x - 4)$$

① Set up a sum and product problem to help factorise the expression.

$$x^2 - 10x + 24$$

$$\_\_ + \_\_ = -10$$
$$\_\_ \times \_\_ = 24$$

② Find two numbers which add to -10 and multiply to 24.

$$(-6) + (-4) = -10$$
$$(-6) \times (-4) = 24$$

③ Use these values to rewrite the expression in the form of $(x + a)(x + b)$.

$$x^2 - 10x + 24$$
$$= (x - 6)(x - 4)$$

Question 118

$(11 + 5x)(11 - 5x)$

① Recall the formula for the **difference of two squares.**

$$a^2 - b^2 \equiv (a + b)(a - b)$$

② Use this formula to factorise $121 - 25x^2$

$$121 - 25x^2 = 11^2 - (5x)^2$$
$$= (11 + 5x)(11 - 5x)$$

Question 119

$(2x + 5)(2x - 1)$

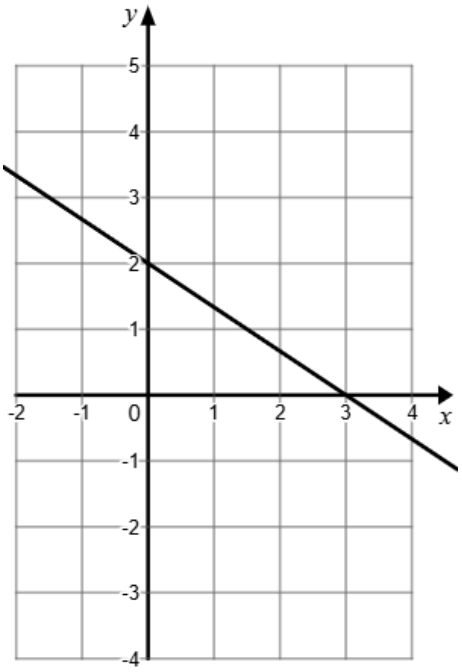
① Find two numbers that add to give 8 and multiply to give $4 \times -5 = -20$.

$$-2 + 10 = 8$$
$$-2 \times 10 = -20$$

② Split the middle term $8x$ into $-2x + 10x$. Then factorise the first two terms and last two terms.

$$4x^2 + 8x - 5$$
$$= 4x^2 - 2x + 10x - 5$$
$$= 2x(2x - 1) + 5(2x - 1)$$
$$= (2x + 5)(2x - 1)$$

Question 120



① Identify the y-intercept and treat this as the starting point.

$$\text{Point}_{start} = (0,2)$$

② As the gradient is $-\frac{2}{3}$, for every 3 units across in the positive horizontal direction go -2 units in the vertical direction.

$$\begin{aligned}\text{Point}_{new} &= (0 + 3, 2 - 2) \\ &= (3, 0)\end{aligned}$$

Question 121

$3x + 2y - 6 = 0$

Question 122

Not parallel

Question 123

$y = \frac{1}{5}x - \frac{244}{5}$ or $y = \frac{1}{5}x - \frac{244}{5}$

Question 124

$20z$

Let z be Adam's age.

Latika is $5z$ year-old.

John is $20z$ year-old.

Question 125

$10x^2 - 6x$

Question 126

$y = -12$

① Subtract 52 from both sides.

$$\begin{array}{r} 4 = 52 + 4y \\ -52 \downarrow \quad \downarrow -52 \\ -48 = 4y \end{array}$$

② Divide both sides by 4.

$$\begin{array}{r} -48 = 4y \\ \div 4 \downarrow \quad \downarrow \div 4 \\ -12 = y \end{array}$$

③ Conclude that $y = -12$.

Question 127

$x = -4$

① Add $\frac{1}{4}x$ to both sides.

$$\begin{array}{r} 6 - \frac{1}{4}x = 7 \\ +\frac{1}{4}x \downarrow \quad \downarrow +\frac{1}{4}x \\ 6 = 7 + \frac{1}{4}x \end{array}$$

② Solve the resulting equation.

$$\begin{array}{r} 6 = 7 + \frac{1}{4}x \\ -7 \downarrow \quad \downarrow -7 \\ -1 = \frac{1}{4}x \\ \times 4 \downarrow \quad \downarrow \times 4 \\ -4 = x \end{array}$$

Question 128

$x = -26$

① Multiply both sides by 3 and 2 to eliminate the fractions.

$$\begin{array}{c} \frac{7x-4}{3} = \frac{5x+6}{2} \\ \times 3 \downarrow \quad \downarrow \times 3 \\ 7x - 4 = \frac{5x+6}{2} \times 3 \\ \times 2 \downarrow \quad \downarrow \times 2 \\ 2(7x - 4) = (5x + 6) \times 3 \\ 2(7x - 4) = 3(5x + 6) \\ 14x - 8 = 15x + 18 \end{array}$$

② Subtract $14x$ from both sides.

$$\begin{array}{c} 14x - 8 = 15x + 18 \\ -14x \downarrow \quad \downarrow -14x \\ -8 = x + 18 \end{array}$$

③ Solve the resulting equation.

$$\begin{array}{c} -8 = x + 18 \\ -18 \downarrow \quad \downarrow -18 \\ -26 = x \end{array}$$

Question 129

$x = \frac{41}{2}$

① Expand the brackets and simplify.

$$\begin{array}{l} 2(x-7) - 8 = 19 \\ 2x - 14 - 8 = 19 \\ 2x - 22 = 19 \end{array}$$

② Solve.

$$\begin{array}{c} 2x - 22 = 19 \\ +22 \downarrow \quad \downarrow +22 \\ 2x = 41 \\ \div 2 \downarrow \quad \downarrow \div 2 \\ x = \frac{41}{2} \end{array}$$

Question 130

$x = 3$

① Find an algebraic expression for the volume.

$$5 \times (3 + 2x) \times 5$$
$$= 25(3 + 2x)$$

② Equate this expression to 225 then solve.

$$\begin{aligned} 25(3 + 2x) &= 225 \\ \div 25 \downarrow \quad & \quad \downarrow \div 25 \\ 3 + 2x &= 9 \\ -3 \downarrow \quad & \quad \downarrow -3 \\ 2x &= 6 \\ \div 2 \downarrow \quad & \quad \downarrow \div 2 \\ x &= 3 \end{aligned}$$

Question 131

$135 - 6x$

Question 132

$x = 3$ or $x = 6$

① Factorise.

$$x^2 - 9x + 18 = 0$$
$$(x - 3)(x - 6) = 0$$

② Write the first factor equal to zero and solve.

$$\begin{aligned} (x - 3)(x - 6) &= 0 \\ x - 3 &= 0 \\ x &= 3 \end{aligned}$$

③ Write the second factor equal to zero and solve.

$$\begin{aligned} (x - 3)(x - 6) &= 0 \\ x - 6 &= 0 \\ x &= 6 \end{aligned}$$

④ Conclude.

$x = 3$ or $x = 6$

Question 133

$x = -2$ or $x = 2$

Question 134

$x = 2$ or $x = 1$

Method 1: by completing the square.

$$\begin{aligned} 3x^2 - 9x + 6 &= 0 \\ 3x^2 - 9x &= -6 \\ x^2 - 3x &= -2 \\ \left(x - \tfrac{3}{2}\right)^2 - \tfrac{9}{4} &= -2 \\ \left(x - \tfrac{3}{2}\right)^2 &= \tfrac{1}{4} \\ x - \tfrac{3}{2} &= \pm \tfrac{1}{2} \\ x &= 2 \text{ or } 1 \end{aligned}$$

Putting this into a calculator, we find that to 2 decimal places,

$x = 2$ or 1

Method 2: using the quadratic formula.

Using $a = 3$, $b = -9$ and $c = 6$

$$x = \frac{9 \pm \sqrt{(-9)^2 - 4 \times 3 \times 6}}{2 \times 3}$$

Putting this into a calculator, we find that to 2 decimal places,

$x = 2$ or 1

Question 135

$x = 10 + \sqrt{19}$ or $x = 10 - \sqrt{19}$

① Write down the equation in completed square form.

$$(x - 10)^2 - 19 = 0$$

② Add 19 to both sides of the equation.

$$\begin{aligned} (x - 10)^2 - 19 &= 0 \\ {\scriptstyle +19 \downarrow} \quad & \quad {\scriptstyle \downarrow +19} \\ (x - 10)^2 &= 19 \end{aligned}$$

③ Square root each side of the equation.

$$(x-10)^2 = 19$$
$$\sqrt{\downarrow} \quad \downarrow\sqrt{}$$
$$x-10 = \pm\sqrt{19}$$

④ Add 10 from both sides of the equation.

$$x-10 = \pm\sqrt{19}$$
$$+10\downarrow \quad \downarrow +10$$
$$x = 10 \pm \sqrt{19}$$

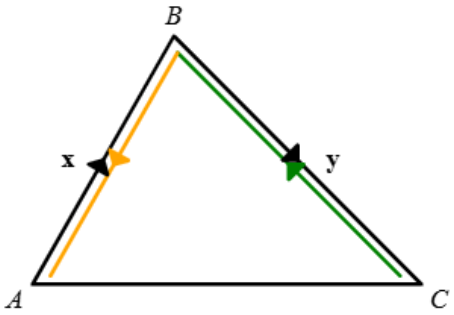
Question 136

$\frac{1}{2}$

Question 137

$-y-x$

① Follow a path from C to A, like the path shown below.



② The edge $\overrightarrow{CB}$ is the edge labelled as y, but note that it is in the reverse direction from the original so describe it as $-y$.

③ The edge $\overrightarrow{BA}$ is the edge labelled as x, but note that it is in the reverse direction from the original so describe it as $-x$.

④ Add the components together.

$$\text{Path } \overrightarrow{CA} = -y-x$$

Question 138

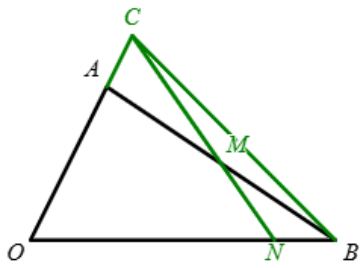
$\frac{1}{2}(a+b)$

Question 139

The vectors are not parallel

Question 140

4: 1



① Find $\overrightarrow{CM}$.

$$\begin{aligned}\overrightarrow{CM} &= \overrightarrow{CA} + \overrightarrow{AM} \\ &= -\frac{1}{3}a + \frac{1}{2}(b-a) \\ &= -\frac{5}{6}a + \frac{1}{2}b\end{aligned}$$

② Find $\overrightarrow{ON}$.

$$\begin{aligned}\overrightarrow{ON} &= \overrightarrow{OC} + \overrightarrow{CN} \\ &= \overrightarrow{OC} + \lambda\,\overrightarrow{CM} \\ &= \frac{4}{3}a + \lambda(-\frac{5}{6}a + \frac{1}{2}b) \\ &= (\frac{4}{3} - \frac{5}{6}\lambda)a + \frac{1}{2}\lambda b\end{aligned}$$

③ Equate the coefficient of a to 0.

$$\begin{aligned}\frac{4}{3} - \frac{5}{6}\lambda &= 0 \\ \lambda &= \frac{8}{5}\end{aligned}$$

④ Find $\overrightarrow{ON}$ and the ratio.

$$\begin{aligned}\overrightarrow{ON} &= \frac{8}{5} \times \frac{1}{2}b \\ &= \frac{4}{5}b\end{aligned}$$

$$\therefore ON : NB = 4 : 1$$

Question 141

24

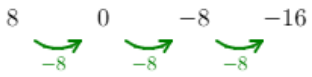
① Substitute $n = 9$ into $3n - 3$

$$\begin{aligned} 9\text{th term} &= 3 \times 9 - 3 \\ &= 24 \end{aligned}$$

Question 142

$16 - 8n$

① Find the common difference.



② The common difference is -8, so we consider the sequence with nth term of $-8n$.

$-8, -16, -24, -32, \ldots$

③ The first term of the sequence with this formula is -8, but the first term of the original sequence is $16 - 8n$. So an adjustment of $+16$ is required.

Therefore, the nth term is $16 - 8n$.

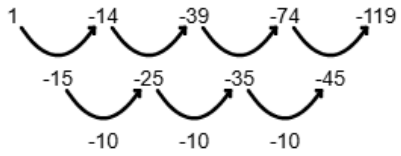
Question 143

$13p - 16q$

Question 144

$-5n^2 + 6$

① Find the first and second differences of the quadratic sequence.



② Halve the constant second difference to get the coefficient of n^2.

$$\frac{-10}{2} = -5 \rightarrow -5\,n^2$$

③ Write down the first few terms of the sequence represented by an nth term of $-5n^2$.

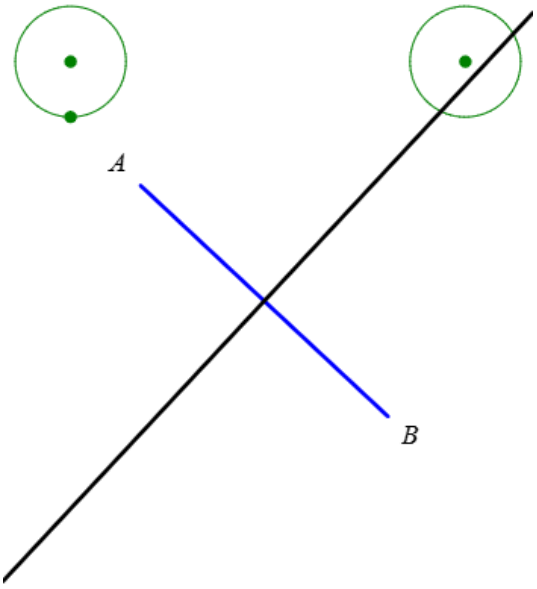
-5	-20	-45	-80	-125

④ Inspect the adjustment needed to get from these numbers to the sequence.

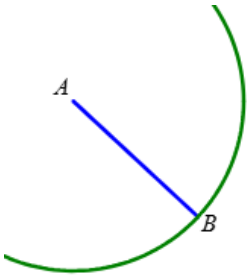
-5	-20	-45	-80	-125
+6	+6	+6	+6	+6
1	-14	-39	-74	-119

⑤ These differences form a linear sequence with an nth term of $-5n^2 + 6$. Add this to the expression found in Step ② to get an nth term of $-5n^2 + 6$.

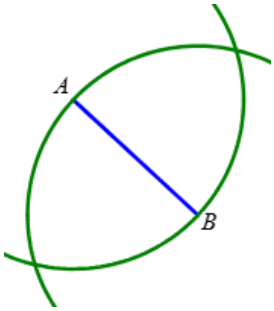
Question 145



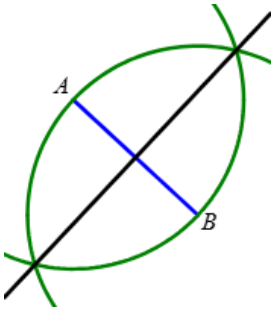
Step 1: put the needle point of the compass on A and draw an arc using AB as the radius.



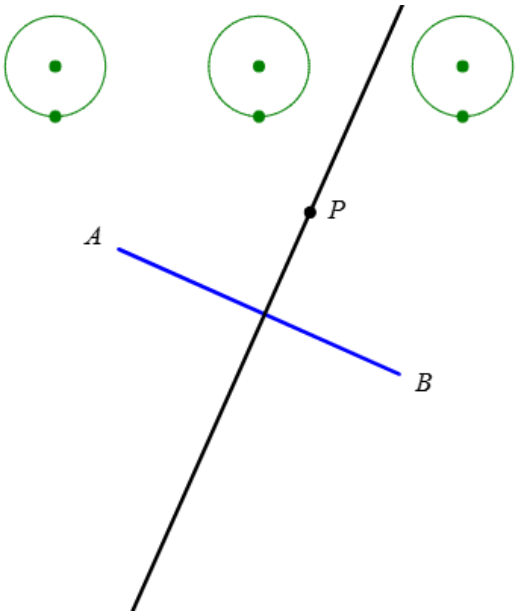
Step 2: put the needle point of the compass on B and draw an arc using AB as the radius.



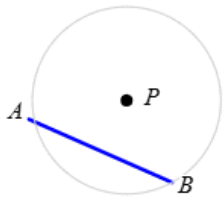
Step 3: join the two intersections with a straight line.



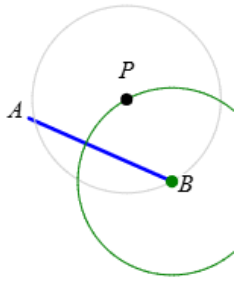
Question 146



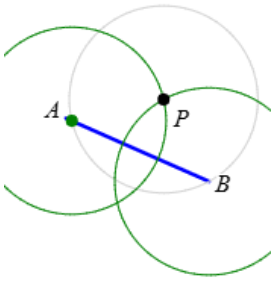
Step 1: put the needle point of the compass on P and draw an arc cutting AB twice.



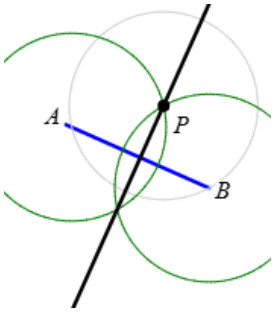
Step 2: put the needle point on one of the intersections and draw an arc passing through P.



Step 3: put the needle point on the other intersection and draw an arc passing through P.



Step 4: join P with the intersection of the last two arcs.



Question 147

Option 4

Question 148

Option 2