Mark scheme

GCSE AQA Higher Tier Paper 2 & 3 Likely Topics - 2025

Question 1

$A = \{$ **3** $\}$
$B = \{$ **5, 11 OR 5,11 OR 5 11** $\}$
$C = \{$ **1** $\}$
$D = \{$ **2, 7, 10 OR 2,7,10 OR 2 7 10** $\}$

① $A = \{3\}$ consisting of the elements that are in X and not in Y

$\xi = \{1,2,3,5,7,10,11\}$
$X = \{3,5,11\}$
$Y = \{1,5,11\}$

② $B = \{5,11\}$ consisting of the elements that are in X and Y

$\xi = \{1,2,3,5,7,10,11\}$
$X = \{3,5,11\}$
$Y = \{1,5,11\}$

③ $C = \{1\}$ consisting of the elements that are in Y and not in X

$\xi = \{1,2,3,5,7,10,11\}$
$X = \{3,5,11\}$
$Y = \{1,5,11\}$

④ $D = \{2,7,10\}$ consisting of the elements that are not in X and not in Y

$\xi = \{1,2,3,5,7,10,11\}$
$X = \{3,5,11\}$
$Y = \{1,5,11\}$

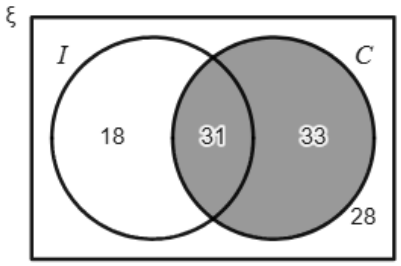
Question 2

$a = 15 - x, b = 16 - x, c = 6$

Question 3

$\frac{64}{110}$

① Select the appropriate parts of the Venn Diagram.



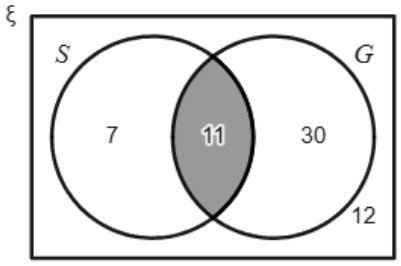
② The probability is the number in the selected regions divided by the total.

$$\text{Probability} = \frac{64}{110}$$

Question 4

$\frac{11}{60}$

① Select the relevant regions of the Venn diagram.



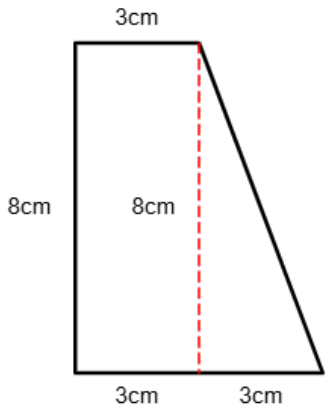
② Calculate $P(S \cap G)$

$$P(S \cap G) = \frac{11}{60}$$

Question 5

180cm ^{3}

① Split the cross section into a rectangle and triangle.



② Work out the area of the rectangle.

$$A = 8 \times 3 = 24 \text{ cm}^2$$

③ Work out the area of the triangle.

$$A = \frac{8 \times 3}{2} = 12 \text{ cm}^2$$

④ Add together the two areas.

$$A = 24 + 12 = 36 \text{ cm}^2$$

⑤ Multiply the area of the cross section by the length of the prism to find the volume.

$$V = 36 \times 5 = 180 \text{ cm}^3$$

Question 6

$48q^7$

① Calculate the area of the cross section by multiplying together the expressions for the width and height and dividing by 2.

$$(4q^3 \times 8q^3) \div 2 = 16q^6$$

② Multiply the expression for the area by the expression for the depth.

$$16q^6 \times 3q = 48q^7$$

Question 7

2511π cm ^{3}

Question 8

x =3.5cm

Question 9

£2665.44

① Calculate the decimal multiplier equivalent to an increase of $1.25\ \%$

$$\frac{100 + 1.25}{100} = 1.0125$$

② Use the formula for compound interest to calculate the final amount.

$$2325 \times 1.0125^{11} = 2665.4362\ \ldots$$
$$\approx pound\ 2665.44$$

Question 10

£1014.55

① Apply the decimal multipliers for each percentage change.

$$1200 \times 0.97^4 \times 0.955^1$$
$$= 1014.55$$

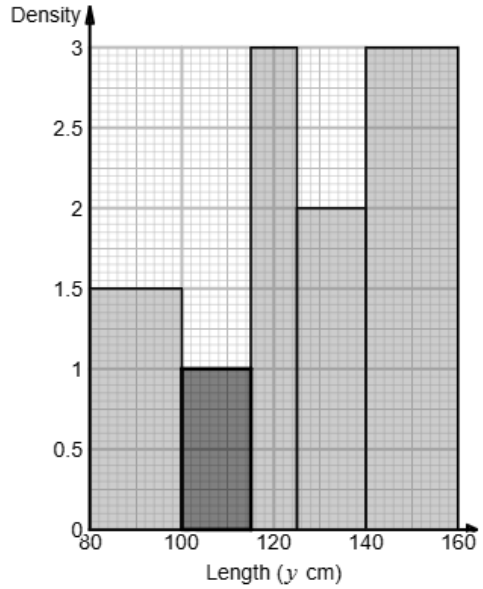
Question 11

Account B

Question 12

$r = 3.09$

Question 13



The frequency density is $\frac{\text{frequency}}{\text{class width}}$.

Therefore the density for the class $100 < y \leq 115$ is:

$$\frac{15}{115-100} = \frac{15}{15} = 1$$

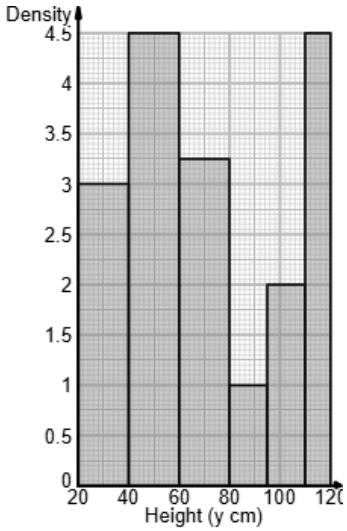
Question 14

155plants

Given that the frequency is 45 for the bar between 110 and 120, the frequency density is therefore:

$$\frac{45}{120-110} = \frac{45}{10} = 4.5$$

The frequency density scale can thus be deduced:



The frequency for the bar between 40 and 60 is $20 \times 4.5 = 90$

The frequency for the bar between 60 and 80 is $20 \times 3.25 = 65$

The total frequency required is 155.

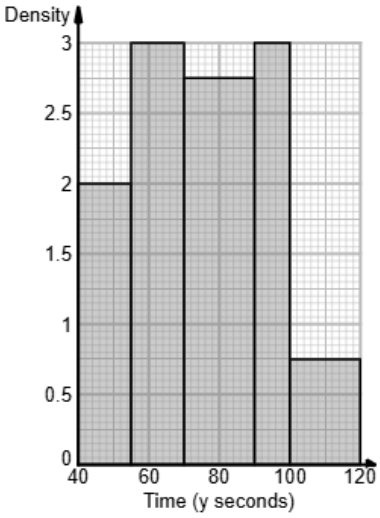
Question 15

175athletes

Given that the frequency is 15 for the bar between 100 and 120, the frequency density is therefore:

$$\frac{15}{120 - 100} = \frac{15}{20}$$
$$= 0.8$$

The frequency density scale can thus be deduced:



The frequency is equal to the frequency density multiplied by the class width.

Time (y seconds)	Frequency
$40 < y \leq 55$	$15 \times 2 = 30$
$55 < y \leq 70$	$15 \times 3 = 45$
$70 < y \leq 90$	$20 \times 2.75 = 55$
$90 < y \leq 100$	$10 \times 3 = 30$
$100 < y \leq 120$	$20 \times 0.75 = 15$

Therefore the total frequency is 175.

Question 16

866250

① Multiply the amount of teachers, boys and girls together.
The number of girls is given by $400 - 175 = 225$.

$$22 \times 175 \times 225 = 866250$$

Question 17

7920

① Multiply together the numbers of choices for each position.

There are 11 choices for the first place finisher.

Then, there are 10 choices for the second place finisher and so on.

$$\begin{aligned}&\text{Number of combinations}\\&= 11 \times 10 \times 9 \times 8\\&= 7920\end{aligned}$$

Question 18

190combinations

① Use the formula $\frac{n(n-1)}{2}$

There are 20 choices for the first singer and 19 choices for the second.

However, this value has be halved as either singer can be picked first.

$$\begin{aligned}\text{Combinations} &= \frac{20 \times 19}{2}\\&= 190\end{aligned}$$

Question 19

$\frac{672}{2744}$

① Work out the denominator by working out the total number of choices.

For each button press there are 14 choices.

$$\begin{aligned}&\text{Number of codes}\\&= 14 \times 14 \times 14\\&= 14^3\\&= 2744\end{aligned}$$

② To work out the numerator, first consider the combinations for letter, number, letter.

$$\begin{aligned}&\text{Letter} \times \text{Number} \times \text{Letter}\\&= 8 \times 6 \times 8\\&= 384\end{aligned}$$

③ Consider the combinations for number, letter, number.

$$\begin{aligned}&\text{Number} \times \text{Letter} \times \text{Number}\\&= 6 \times 8 \times 6\\&= 288\end{aligned}$$

④ Add the two answers together.

$$\begin{aligned}&384 + 288\\&= 672\end{aligned}$$

⑤ Write the answer as a fraction.

$\frac{672}{2744}\left(=\frac{12}{49}\right)$

Question 20

28red balls
16green balls

① Calculate the total parts in the ratio.

$$7 + 4 = 11 \text{ parts}$$

② Determine the value of one part in the ratio.

$$44 \div 11 = 4$$

③ Multiply the ratio by the value of one part.

$$\begin{array}{c} 7:4 \\ \times 4\downarrow \quad \downarrow \times 4 \\ 28:16 \end{array}$$

There are 28 red balls.
There are 16 green balls.

Question 21

£6

① Determine how many parts in the ratio corresponds to £24' />.

$$\begin{array}{c} \text{Celeste} : \text{Jin} : \text{Ursula} \\ 1:2:4 \end{array}$$

$£24$ corresponds to 4 parts in the ratio.

② Calculate the value of one part in the ratio.

$$£24 \div 4 = £6$$

③ Calculate the difference in number of parts for Celeste and Jin.

$$2 - 1 = 1$$

④ Calculate the value of 1 parts.

$$1 \times £6 = £6$$

Question 22

297males over 50

① Share 900 in the ratio $3:2$

$$\begin{array}{rcl} 3:2 & = & 5 \text{ parts} \\ \times 180\downarrow & & \downarrow \times 180 \\ 540:360 & = & 900 \end{array}$$

② Find $\frac{9}{20}$ of 540.

$$540 \times \frac{9}{20} = 243$$

③ Subtract 243 from 540.

$$540 - 243 = 297$$

Question 23

132miles

① Calculate the scale factor by dividing the final length of time by the initial length of time then simplifying the resulting fraction.

$$\begin{aligned}\text{scale factor} &= 77 \div 63 \\ &= \frac{11}{9}\end{aligned}$$

② A scale factor of $\frac{11}{9}$ is equivalent to multiplying by 11 and dividing by 9.

③ Write a ratio table and use the scale factor to find the required distance.

63minutes	$\overset{\div 9}{\rightarrow}$	7minutes	$\overset{\times 11}{\rightarrow}$	77minutes
108miles	$\overset{\div 9}{\rightarrow}$	12miles	$\overset{\times 11}{\rightarrow}$	132miles

Question 24

Exactly enough

Question 25

3classrooms

① Work out how many classrooms would be needed if the class size was 1.

$$\begin{array}{cc} 27 \text{ pupils} & 2 \text{ classrooms} \\ \div 27 \downarrow & \downarrow \times 27 \\ 1 \text{ pupil} & 54 \text{ classrooms} \end{array}$$

② Now work out how classrooms would be needed if the class size was 18 .

$$\begin{array}{cc} 1 \text{ pupil} & 54 \text{ classrooms} \\ \times 18 \downarrow & \downarrow \div 18 \\ 18 \text{ pupils} & 3 \text{ classrooms} \end{array}$$

Question 26

$\frac{1}{4}y^2$

① Writing a proportionality formula where the constant is k.

$$k = k \times y^2$$

② Substitute the given values to work out k.

$$36 = k \times 12^2$$
$$\frac{36}{144} = k$$
$$k = \frac{1}{4}$$

③ Substitute $k = \frac{1}{4}$ into $k = k \times y^2$.

$$k = \frac{1}{4}y^2$$

Question 27

$\frac{16}{y}$

① Writing a proportionality formula where the constant is k.

$$p = \frac{k}{y}$$

② Substitute the given values to work out k.

$$4 = \frac{k}{4}$$
$$4 \times 4 = k$$
$$k = 16$$

③ Substitute $k = 16$ into $p = \frac{k}{y}$.

$$p = \frac{16}{y}$$

Question 28

5

Question 29

$\frac{6\sqrt[3]{c}}{\sqrt[3]{5}}$

① Find an equation for a and b.

$$a = k_1 \times \sqrt[3]{b}$$
$$6 = k_1 \times 3$$
$$k_1 = 2$$
$$\therefore a = 2\sqrt[3]{b}$$

② Find an equation for b and c.

$$b = k_2 \times c$$
$$27 = k_2 \times 5$$
$$k_2 = \frac{27}{5}$$
$$\therefore b = \frac{27}{5}c$$

③ Combine the two equations.

$$a = 2\sqrt[3]{b}$$
$$b = \frac{27}{5}c$$
$$a = 2 \times \sqrt[3]{\frac{27}{5}c}$$
$$a = \frac{6\sqrt[3]{c}}{\sqrt[3]{5}}$$

Question 30

32

① Construct an equation linking m and n.

$$m \propto \sqrt[3]{n}$$
$$\therefore m = k\sqrt[3]{n}$$

② Find an expression for k in terms of d.

$$m = 64 \text{ when } n = 8d$$
$$\Rightarrow 64 = k \times \sqrt[3]{8d}$$
$$\Rightarrow 64 = k \times 2\sqrt[3]{d}$$
$$\Rightarrow \frac{64}{2\sqrt[3]{d}} = k$$
$$\Rightarrow \frac{32}{\sqrt[3]{d}} = k$$
$$\therefore m = \frac{32}{\sqrt[3]{d}}\sqrt[3]{n}$$

③ Substitute $n = d$ into $m = \frac{32}{\sqrt[3]{d}}\sqrt[3]{n}$

$$m = \frac{32}{\sqrt[3]{d}}\sqrt[3]{d}$$
$$\Rightarrow m = \frac{32}{\cancel{\sqrt[3]{d}}}\cancel{\sqrt[3]{d}}$$
$$\Rightarrow m = 32$$

Question 31

x =4.2

① Calculate the value between the two operations.

$$-1.9 + 4 = 2.1$$

② Calculate x.

$$2.1 \times 2 = 4.2$$

Question 32

$\frac{t-3}{5}$

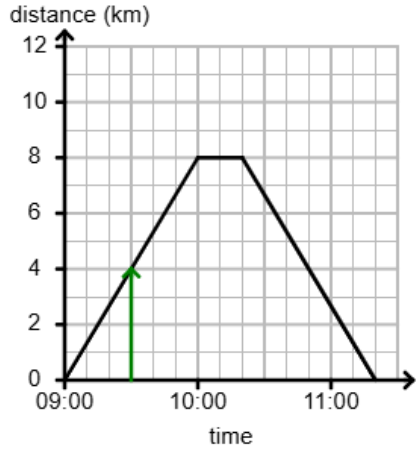
① Find an expression for the value after the first operation.

② Apply the second operation to $t - 3$

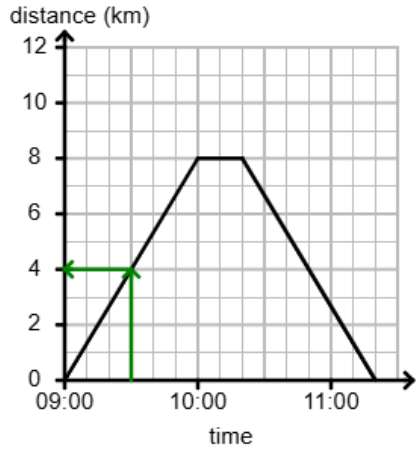
Question 33

4km

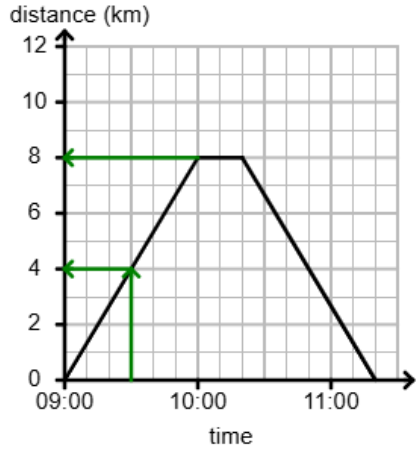
① Draw up from $09{:}30$ on the time axis to meet the distance-time graph.



② Draw left to meet the distance axis.



③ Draw left from the top point of the graph.



④ Find the difference between the two distances.

$8 - 4 \ = \ 4 \text{ km}.$

Question 34

3m/s ^{2}

Question 35

$\frac{59}{111}$

① Write the recurring decimal out to several decimal places and set equal to a variable.

$x = 0.531531 \ldots \quad ①$

② There are three digits recurring so multiply both sides of the equation by 1000.

$1000x \ = \ 531.531531 \ldots \quad ②$

③ Subtract equation ① from equation ② to eliminate the recurring part.

$$999x = 531 \quad ② - ①$$

④ Solve for x.

$$x = \frac{531}{999}$$
$$x = \frac{59}{111}$$

Question 36

$\frac{40}{3}$

Question 37

$3x^2 + 3x$

Question 38

$x = \frac{13}{4}$

① Add 1 to both sides.

$$\begin{array}{c} 4x - 1 = 12 \\ +1\downarrow \quad \downarrow +1 \\ 4x = 13 \end{array}$$

② Divide both sides by 4.

$$\begin{array}{c} 4x = 13 \\ \div 4\downarrow \quad \downarrow \div 4 \\ x = \frac{13}{4} \end{array}$$

Question 39

$x = 4$

① Add $\frac{1}{2}x$ to both sides.

$$\begin{array}{c} 7 = 9 - \frac{1}{2}x \\ +\frac{1}{2}x \downarrow \quad \downarrow +\frac{1}{2}x \\ 7 + \frac{1}{2}x = 9 \end{array}$$

② Solve the resulting equation.

$$\begin{aligned} 7+\tfrac{1}{2}x &= 9 \\ -7 \downarrow \quad & \downarrow -7 \\ \tfrac{1}{2}x &= 2 \\ \times 2 \downarrow \quad & \downarrow \times 2 \\ x &= 4 \end{aligned}$$

Question 40

$x = -18$

① Multiply both sides by 3 to eliminate the fraction.

$$\begin{aligned} \tfrac{11x}{3} &= 4x + 6 \\ \times 3 \downarrow \quad & \downarrow \times 3 \\ 11x &= 3(4x + 6) \\ 11x &= 12x + 18 \end{aligned}$$

② Solve the resulting equation.

$$\begin{aligned} 11x &= 12x + 18 \\ -11x \downarrow \quad & \downarrow -11x \\ 0 &= x + 18 \\ -18 \downarrow \quad & \downarrow -18 \\ -18 &= x \end{aligned}$$

Question 41

11

① Find an algebraic expression for the cost.

$$\begin{aligned} &3 \times 3 + 4 \times x \\ &= 9 + 4x \end{aligned}$$

② Equate this expression to 53 then solve.

$$\begin{aligned} 9 + 4x &= 53 \\ -9 \downarrow \quad & \downarrow -9 \\ 4x &= 44 \\ \div 4 \downarrow \quad & \downarrow \div 4 \\ x &= 11 \end{aligned}$$

Question 42

$x = 400$

Question 43

$x = -\frac{3}{4}$ or $x = \frac{3}{4}$

Question 44

$x = -1.29$ or $x = -2.71$

Method 1: by completing the square.

$$\begin{aligned} 2x^2 + 8x + 7 &= 0 \\ 2x^2 + 8x &= -7 \\ x^2 + 4x &= -\frac{7}{2} \\ (x+2)^2 - 4 &= -\frac{7}{2} \\ (x+2)^2 &= \frac{1}{2} \\ x + 2 &= \pm\frac{\sqrt{2}}{2} \\ x &= -\frac{4+\sqrt{2}}{2} \text{ or } -\frac{4-\sqrt{2}}{2} \end{aligned}$$

Putting this into a calculator, we find that to 2 decimal places,

$x = -1.29$ or -2.71

Method 2: using the quadratic formula.

Using $a = 2$, $b = 8$ and $c = 7$

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 2 \times 7}}{2 \times 2}$$

Putting this into a calculator, we find that to 2 decimal places,

$x = -1.29$ or -2.71

Question 45

y =5.4cm

① Use Pythagoras' theorem.

$$\begin{aligned} a^2+b^2 &= c^2 \\ 5^2+2^2 &= y^2 \\ 29 &= y^2 \\ y &= \sqrt{29} \\ &= 5.385\ldots \\ &= 5.4 \text{ cm} \end{aligned}$$

Question 46

3.6cm

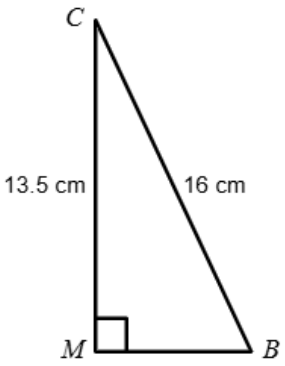
① Use Pythagoras' theorem.

$$\begin{aligned} a^2+b^2 &= c^2 \\ y^2+6^2 &= 7^2 \\ y^2 &= 7^2-6^2 \\ y^2 &= 13 \\ y &= \sqrt{13} \\ &= 3.606\ldots \\ &\approx 3.6\ \text{cm} \end{aligned}$$

Question 47

115.9cm ^{2}

① Use the line of symmetry of the isosceles triangle to draw a right-angled triangle, with the right angle at the midpoint of AB.



② Use Pythagoras' Theorem to find the missing side in this triangle.

$$\begin{aligned} MB\ 2 &= 16\ 2 - 13.5\ ^2 \\ &= 73.75 \\ M\,B &= 8.5878\ \ldots\ cm \end{aligned}$$

③ Find the area of triangle ABC.

$$\begin{aligned} Area &= \tfrac{1}{2} \times 17.1756\ \ldots \times 13.5 \\ &= 115.9351\ \ldots\ cm^2 \end{aligned}$$

④ Round the answer to one decimal place.

$$Area = 115.9\ cm^2$$

Question 48

18.1cm

① Use Pythagoras' theorem in triangle DEF to find the diameter of the semicircle.

$$\begin{aligned} DF\ 2 &= 5^{\,2} + 3.5^{\,2} \\ &= 37.25 \\ DF &= 6.1033\ \ldots \end{aligned}$$

② Find the circumference of the circle and halve it to find the length of the arc DF.

$$\begin{aligned} C &= \pi\, d \\ &= \pi \times 6.1033\ \ldots \\ &= 19.174\ \ldots \\ &\downarrow \ \div 2 \\ arc\ DF &= 9.587\ \ldots \end{aligned}$$

③ Add DE, EF and the arc DF to find the perimeter of the shape.

$$\begin{aligned} P &= 5 + 3.5 + 9.587\ \ldots \\ &= 18.087\ \ldots \end{aligned}$$

④ Round the answer to 1 decimal place.

$$P = 18.1\ cm$$

Question 49

11.7cm

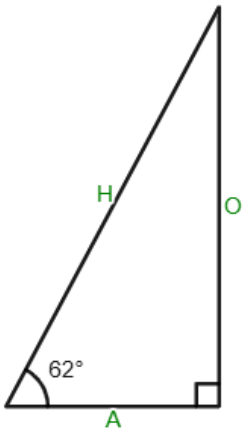
① Use Pythagoras' theorem to find the length EG.

$$\begin{aligned} EG &= \sqrt{10^2+6^2} \\ &= \sqrt{136} \\ &= 11.6619 \\ &= 11.7\ \text{cm} \end{aligned}$$

Question 50

z =15cm

① Label the sides.



② Decide the trigonometric ratio to use.

SOH CAH T**OA**
Therefore we use tan

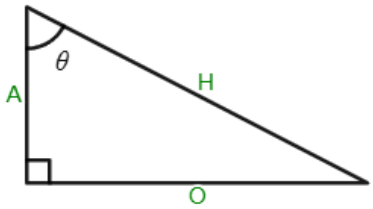
③ Write an equation and solve.

$$\begin{aligned} \tan(\theta) &= \frac{O}{A} \\ \tan(62) &= \frac{z}{8} \\ z &= 8\tan(62) \\ &= 15.0 \text{ cm} \end{aligned}$$

Question 51

$\theta = 62.7^\circ$

① Label the sides.



② Decide the trigonometric ratio to use.

S**OH** CAH TOA
Therefore we use sin

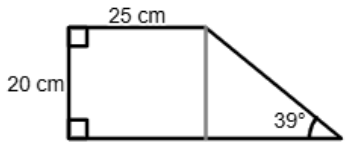
③ Write an equation and solve.

$$\begin{aligned}\sin(\theta) &= \frac{O}{H}\\ \sin(\theta) &= \frac{8}{9}\\ \theta &= \sin^{-1}\left(\frac{8}{9}\right)\\ &= 62.7^{\circ}\end{aligned}$$

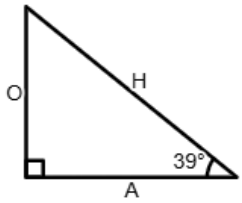
Question 52

126.5cm

① Draw a diagram, creating a right angled triangle.



② Label the triangle and find the adjacent side of the triangle.



$$\begin{aligned}O &= 20\text{ cm}\\ \tan\theta &= \frac{O}{A}\\ A &= \frac{O}{\tan\theta}\\ &= \frac{20}{\tan 39}\\ &= 24.70\ldots\text{ cm}\end{aligned}$$

③ Find the length of the unknown horizontal edge.

$$\begin{aligned}\text{length} &= 25 + 24.70\ldots\\ &= 49.70\ldots\text{ cm}\end{aligned}$$

④ Find the length of the hypotenuse of the triangle.

$$\begin{aligned}\sin\theta &= \frac{O}{H}\\ H &= \frac{O}{\sin\theta}\\ &= \frac{20}{\sin 39}\\ &= 31.78\ldots\text{ cm}\end{aligned}$$

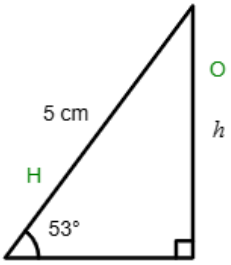
⑤ Add up the lengths of all the sides to find the perimeter.

$$P = 25 + 20 + 49.70\ldots + 31.78\ldots$$
$$\approx 126.5$$

Question 53

θ =44.8 to 45.2 °

① Label the sides.



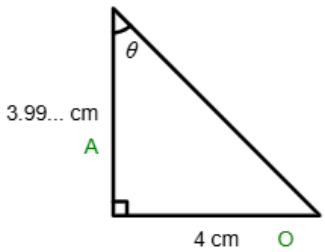
② Decide the trigonometric ratio to use to find h.

SOH CAH TOA
Therefore we use sin

③ Write an equation and solve.

$$\sin(\theta) = \frac{O}{H}$$
$$\sin(53) = \frac{h}{5}$$
$$h = 5 \times \sin(53)$$
$$h = 3.99\ldots$$

④ Label the sides.



⑤ Decide the trigonometric ratio to use.

SOH CAH TOA
Therefore we use tan

⑥ Write an equation and solve.

$$\tan(\theta) = \frac{O}{A}$$
$$\tan(\theta) = \frac{4}{3.99\ldots}$$
$$\theta = \tan^{-1}\left(\frac{4}{3.99\ldots}\right)$$
$$= 45^{\circ}$$

Question 54

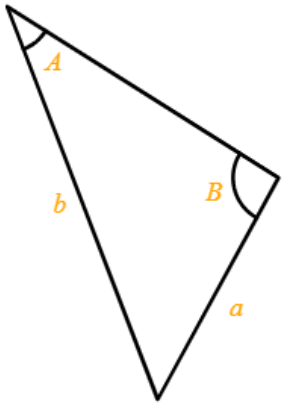
32.8°

Question 55

y =6.6cm

① Recall the formula for the sine rule to find a missing length and label the triangle accordingly.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$



② Substitute the values into the sine rule.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\Rightarrow \frac{y}{\sin(37)} = \frac{11}{\sin(93)}$$

③ Solve for y.

$$\frac{y}{\sin(37)} = \frac{11}{\sin(93)}$$
$$\times \sin(37) \downarrow \quad \downarrow \times \sin(37)$$
$$y = \frac{11}{\sin(93)} \times \sin(37)$$
$$\approx 6.6$$

Question 56

z =102.8°

Taking z'as an acute angle, using the sine rule, $\frac{\sin z'}{12.4} = \frac{\sin(59)}{10.9}$

Multiply both sides by 12.4

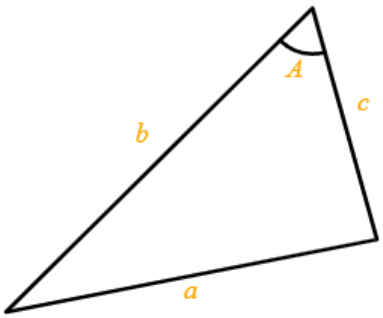
$$\sin z' = \frac{12.4 \times \sin(59)}{10.9}$$
$$z' = \sin^{-1}\left(\frac{12.4 \times \sin(59)}{10.9}\right)$$
$$z' = 77.2^\circ$$
$$z = 180 - 77.2 = 102.8^\circ$$

Question 57

y =7.9cm

① Recall the formula for the cosine rule and label the triangle accordingly.

$$a^2 = b^2 + c^2 - 2bc\cos A$$



② Substitute the values into the cosine rule.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Rightarrow y^2 = 9^2 + 5^2 - 2 \times 9 \times 5 \times \cos(61)$$

③ Calculate and square root.

$$y = \sqrt{9^2 + 5^2 - 2 \times 9 \times 5 \times \cos(61)}$$
$$= \sqrt{62.367\ldots}$$
$$\approx 7.9 \text{ cm}$$

Question 58

11.6cm

Question 59

105.8cm ^{2}

$$\text{Area} = \frac{1}{2} \times 11 \times 22 \times \sin 61$$
$$= 105.8 \text{ cm}^2$$

Question 60

114.9cm ^{2}

• Calculate $\angle ABC$ using the sine rule.

$$\frac{\sin(\angle ABC)}{12} = \frac{\sin(72)}{20}$$
$$\sin(\angle ABC) = \frac{12\sin(72)}{20}$$
$$\angle ABC = \sin^{-1}\left(\frac{12\sin(72)}{20}\right)$$
$$= 34.8$$

• Calculate $\angle CAB$

$$\angle CAB = 180 - 72 - 34.8$$
$$= 73.2$$

• Calculate the area using $\frac{1}{2}ab\sin C$

$$\text{Area} = \frac{1}{2} \times 20 \times 12 \times \sin 73.2$$
$$= 114.9 \text{ cm}^2$$

Question 61

60.5m

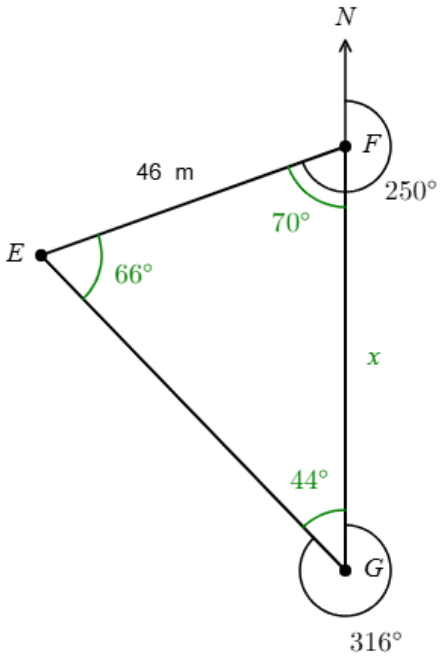
① Find angle $\angle EGF$ and angle $\angle GFE$.

$$\angle EGF = 360 - 316$$
$$= 44^\circ$$

$$\angle GFE = 250 - 180$$
$$= 70^\circ$$

② Find angle $\angle FEG$, using the fact that the angles in a triangle add to 180°.

$$\angle FEG = 180 - 44 - 70$$
$$= 66^\circ$$



③ Use the sine rule to find the length of FG, labelled x on the diagram.

$$\begin{aligned}\frac{x}{\sin(66)} &= \frac{46}{\sin(44)} \\ x &= \frac{46\sin(66)}{\sin(44)} \\ &= 60.494\ \ldots \\ &\approx 60.5m\end{aligned}$$

Question 62

6^{18}

① When raising a power to a power, multiply the indices/exponents.

$$\begin{aligned}\left(6^9\right)^2 &= 6^{9\times 2} \\ &= 6^{18}\end{aligned}$$

Question 63

1

① Anything raised to the power of 0 is equal to 1

$$x^0 = 1$$

Question 64

x^7

① Use laws of indices to simplify.

$$\begin{aligned}\frac{x^2 \times x^9}{x^4} &= \frac{x^{11}}{x^4} \\ &= x^7\end{aligned}$$

Question 65

$3750 \leq x < 3850$

This is as if the number had been rounded to the nearest 100.

To find the lower bound you can subtract half the accuracy, 50, from the rounded number.

To find the upper bound you add half the accuracy.

Lower bound $= 3800 - 50 = 3750$

Upper bound $= 3800 + 50 = 3850$

Therefore the error interval is: $3750 \leq x < 3850$

Question 66

446.75

To find the lower bound of x, use the lower bound of y and the lower bound of z.

$$\begin{aligned}x_{lower} &= 4y_{lower} + z_{lower} \\ &= 4(95.5) + (64.75) \\ &= 446.75\end{aligned}$$

Question 67

0.1

$$\begin{aligned}x_{lower} &= \frac{\sqrt{y_{lower}}}{z_{upper}} \\ &= \frac{\sqrt{0.565}}{7.375} \\ &= 0.10192 \\ x_{upper} &= \frac{\sqrt{y_{upper}}}{z_{lower}} \\ &= \frac{\sqrt{0.575}}{7.365} \\ &= 0.10296\end{aligned}$$

(Could not display math) both bounds agree to that level of accuracy.

Question 68

Length (x cm)	Cumulative Frequency
$10 < x \leq 15$	5
$10 < x \leq 20$	14
$10 < x \leq 25$	47
$10 < x \leq 30$	64
$10 < x \leq 35$	73
$10 < x \leq 40$	100

Length (x cm) Cumulative frequency

$\mathbf{10 < x \leq 15}$ 5

$\mathbf{10 < x \leq 20}$ 14

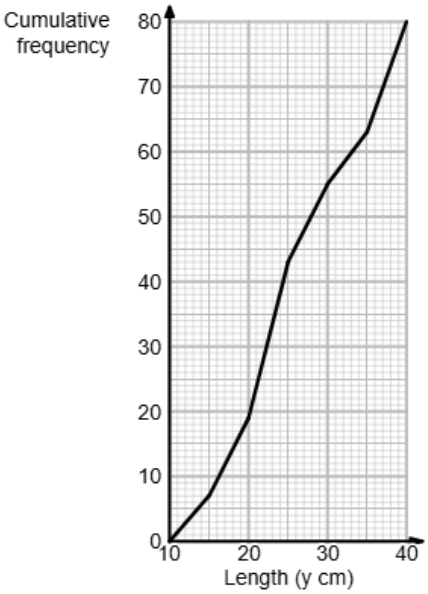
$\mathbf{10 < x \leq 25}$ 47

$\mathbf{10 < x \leq 30}$ 64

$\mathbf{10 < x \leq 35}$ 73

$\mathbf{10 < x \leq 40}$ 100

Question 69



You can draw a cumulative frequency table first.

Length (y cm)	Cumulative frequency
$10 < y \leq 15$	7
$15 < y \leq 20$	19
$20 < y \leq 25$	43
$25 < y \leq 30$	55
$30 < y \leq 35$	63
$35 < y \leq 40$	80

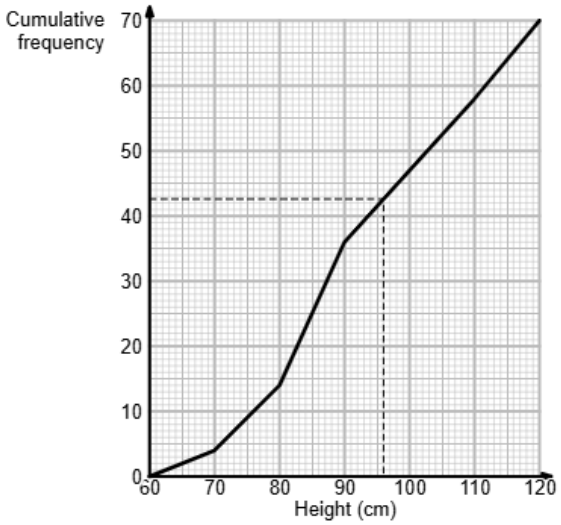
There is no data below 10 so you can plot this with a cumulative frequency of 0.

You then need to plot the end of each class interval with its cumulative frequency.

Question 70

flowers

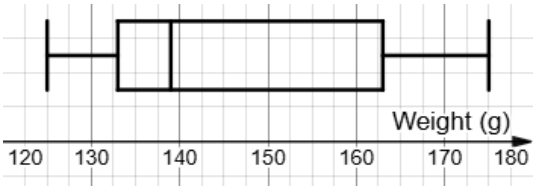
① Find the point on the graph where the height is 96 cm.

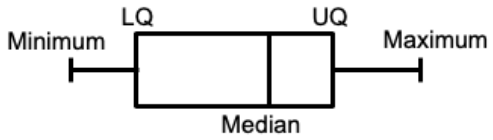


③ Conclude.

The answer is 27.4.

Question 71





Question 72

$\binom{25}{17}$

① Multiply each vector by its scalar.

$$\begin{aligned} 2a - 2c + 3b &= 2\begin{pmatrix}4\\7\end{pmatrix} - 2\begin{pmatrix}-7\\6\end{pmatrix} + 3\begin{pmatrix}1\\5\end{pmatrix} \\ &= \begin{pmatrix}2\times 4\\2\times 7\end{pmatrix} + \begin{pmatrix}-2\times -7\\-2\times 6\end{pmatrix} + \begin{pmatrix}3\times 1\\3\times 5\end{pmatrix} \\ &= \begin{pmatrix}8\\14\end{pmatrix} + \begin{pmatrix}14\\-12\end{pmatrix} + \begin{pmatrix}3\\15\end{pmatrix} \end{aligned}$$

② Add the vectors together.

$$\begin{aligned} &= \begin{pmatrix}8+14+3\\14-12+15\end{pmatrix} \\ &= \begin{pmatrix}25\\17\end{pmatrix} \end{aligned}$$

Question 73

$10a + 4b$

Question 74

$6y^2 - 9y - 15p\ y$

① Multiply all terms in the bracket by $3y$, and then simplify.

$$\begin{aligned} &3y\ (2y - 3 - 5p) \\ &= 3y \times 2y + 3y \times -3 + 3y \times -5p \\ &= 6y^2 - 9y - 15p\ y \end{aligned}$$

Question 75

$27x^2 - 47x$

① Expand each bracket.

$$\begin{aligned} &6x(7x - 2) - 5x(3x + 7) \\ &= 42x^2 - 12x - 15x^2 - 35x \end{aligned}$$

② Collect like terms and simplify.

$$42x^2 - 12x - 15x^2 - 35x$$
$$= 42x^2 - 15x^2 - 12x - 35x$$
$$= 27x^2 - 47x$$

Question 76

$x^2 - 10x + 25$

① Multiply each term in the first bracket by each term in the second bracket.

$$(x - 5)^2$$
$$= (x + -5)(x + -5)$$
$$= x \times x + x \times -5 + -5 \times x + -5 \times -5$$
$$= x^2 - 5x - 5x + 25$$

② Simplify.

$$= x^2 - 10x + 25$$

Question 77

$-20a^3 + 76a^2 - 73a + 20$

① Expand the first two brackets.

$$(5a - 4)(2a - 1)(5 - 2a)$$
$$= \left(10a^2 - 5a - 8a + 4\right)(5 - 2a)$$
$$= \left(10a^2 - 13a + 4\right)(5 - 2a)$$

② Expand the remaining two brackets.

$$\left(10a^2 - 13a + 4\right)(5 - 2a)$$
$$= 50a^2 - 20a^3 - 65a + 26a^2 + 20 - 8a$$
$$= -20a^3 + 76a^2 - 73a + 20$$

Question 78

$3h(10h + 1)$

Question79

$(x + 4)(3x - 2)$

① Find two numbers that add to give 10 and multiply to give $3 \times -8 = -24$.

$$-2 + 12 = 10$$
$$-2 \times 12 = -24$$

② Split the middle term $10x$ into $-2x + 12x$. Then factorise the first two terms and last two terms.

$$\begin{aligned} & 3x^2 + 10x - 8 \\ = \ & 3x^2 - 2x + 12x - 8 \\ = \ & x(3x - 2) \ + \ 4(3x - 2) \\ = \ & (x \ + \ 4)(3x - 2) \end{aligned}$$

Question 80

$8n$

① Find the common difference.

② The common difference is $+8$, so we consider the sequence with nth term of $8n$.

$8,16,24,32, \ldots$

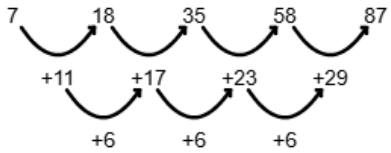
③ The first term of the sequence with this formula is $8n$, but the first term of the original sequence is $8n$. So an adjustment of is required.

Therefore, the nth term is $8n$.

Question 81

$3n^2 + 2n + 2$

① Find the first and second differences of the quadratic sequence.



② Halve the constant second difference to get the coefficient of n^2.

$\frac{6}{2} = 3 \rightarrow 3n^2$

③ Write down the first five terms of the sequence represented by an nth term of $3n^2$.

3 12 27 48 75

④ Inspect the adjustment needed to get from these numbers to the sequence.

3	12	27	48	75
+4	+6	+8	+10	+12
7	18	35	58	87

⑤ These differences form a linear sequence with an nth term of $2n + 2$. Add this to the expression found in Step ② to get an nth term of $3n^2 + 2n + 2$.

Question 82

1400

① Write both numbers as product of prime factors.

$$100 = 2^2 \times 5^2$$
$$56 = 2^3 \times 7$$

② Write down each prime factor with the larger index.

$$2^3 \times 5^2 \times 7$$

③ Calulate the Lowest Common Multiple.

$$2^3 \times 5^2 \times 7 \ = \ 1400$$

Question 83

6

① Write all numbers as product of prime factors.

$$54 \ = \ 2 \times 3^3$$
$$72 \ = \ 2^3 \times 3^2$$
$$30 \ = \ 2 \times 3 \times 5$$
$$12 \ = \ 2^2 \times 3$$

② Find the prime factors in common.

$$2 \times 3$$

③ Conclude.

The Highest Common Factor is 6

Question 84

20.7cm

① Find the length of the arc.

$$l = \frac{123}{360} \times 2\pi \times 5$$
$$= 10.7338$$

② Find the perimeter.

$$P = 10.7338 + 2 \times 5$$
$$= 20.7 \text{ cm}$$

Question 85

q =3.3cm

The area of a sector is $\frac{\theta}{360} \times \pi r^2$ where r is the radius and θ the angle of the sector.

Substituting gives

$$6 = \frac{65}{360} \times \pi \times q^2$$

Dividing both sides by $\frac{65}{360} \times \pi$ gives

$$\frac{6}{\frac{65}{360} \times \pi} = q^2$$

Square rooting both sides:

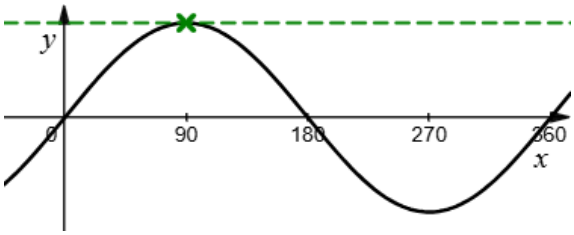
$$q = \sqrt{\frac{6}{\frac{65}{360} \times \pi}}$$

Therefore $q = 3.3$ cm

Question 86

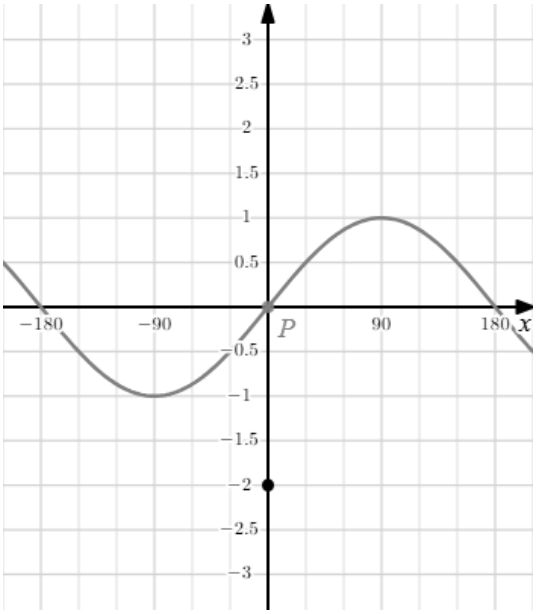
1solution(s)

① Draw a horizontal line at $y = 1$ and indicate any solutions for $90^\circ \leq x \leq 360^\circ$.



② There is only 1 solution within this range.

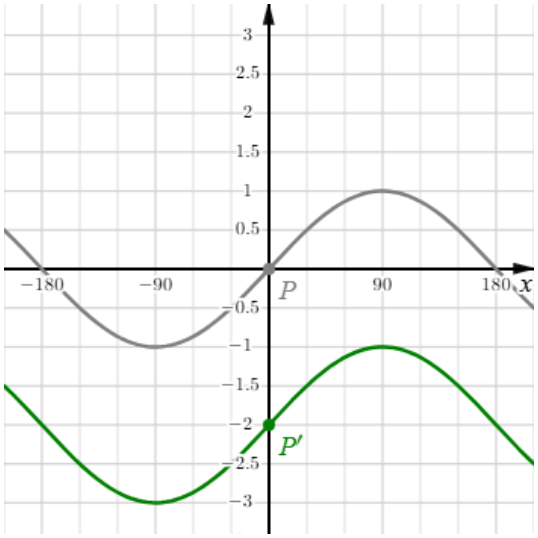
Question 87



① Subtracting 2 from the outside of the function translates the graph by the vector $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$, this can be achived by subtracting 2 from all of the y-coordinates.

$$P(0,0) \overset{(\,,-2)}{\rightarrow} P'(0,-2)$$

② Sketch the translated function.



Question 88

$2n^2 + 6n + 9$

① Expand and simplify.

$$(2n + 3)^2 + 10$$
$$= (2n + 3)(2n + 3) + 10$$
$$= 4n^2 + 6n + 6n + 9 + 10$$
$$= 4n^2 + 12n + 19$$

② Rewrite into the form $2(\ldots) + 1$

$$= 4n^2 + 12n + 18 + 1$$
$$= 2(2n^2 + 6n + 9) + 1$$

Therefore it is an odd number for all integer values of n.

Question 89

$4y + 4$ or $-4y - 4$

① Expand the brackets and simplify

$$(3y + 1)(5y + 9) + (y + 1)(y + 7) - 8y$$
$$= 15y^2 + 32y + 9 + y^2 + 8y + 7 - 8y$$
$$= 16y^2 + 32y + 16$$

② Show that $16y^2 + 32y + 16$ is a perfect square.

$$(ay+b)^2 = 16y^2+32y+16$$
$$a^2y^2+2aby+b^2 = 16y^2+32y+16$$

$$y^2 \text{ terms} \Rightarrow a^2 = 16$$
$$a = \pm 4$$
$$\text{number terms} \Rightarrow b^2 = 16$$
$$b = \pm 4$$

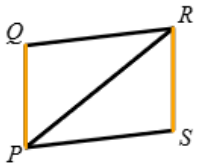
Since all terms are positive, take a and b to both be positive and verify this satisfies $2ab = 32$.

$$2 \times 4 \times 4 = 32 \ \checkmark$$
$$\therefore (4y+4)^2 = 16y^2+32y+16$$

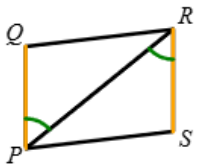
Question 90

$PQ =$ **RS OR SR** .
Angle $QPR =$ Angle **PRS OR SRP** .
Reason: **alternate** .
Triangles PQR and PRS share side **PR OR RP** .
Therefore using congruence condition **SAS** , triangles PQR and PRS are congruent.

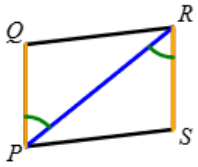
① Identify equal sides PQ and RS.



② Identify that alternate angles QPR and PRS are equal.



③ Identify that both triangles share length PR.



④ Use the congruence condition SAS to conclude triangles PQR and PRS are congruent.