"Full Coverage": Algebraic Proofs involving Integers

This worksheet is designed to cover one question of each type seen in past papers, for each GCSE Higher Tier topic. This worksheet was automatically generated by the DrFrostMaths Homework Platform: students can practice this set of questions interactively by going to www.drfrostmaths.com/homework, logging on, *Practise → Past Papers/Worksheets* (or *Library → Past/Past Papers* for teachers), and using the 'Revision' tab.

Question 1

Categorisation: Identify the type of number indicated by an algebraic expression, e.g. $2n, 2n+1, 4n$.

[Edexcel GCSE Nov2015-2F Q13a]

n is a positive whole number.

What type of positive whole number is $2n-1$?

.....

Question 2

Categorisation: Determine properties of resulting integers when odd/even numbers are added, subtracted or multipied.

[Edexcel GCSE June2007-6H Q4 Edited]

Seejal says

"If a and b are prime numbers greater than 2, then a × b is always an odd number."

Is Seejal correct?

[ ] Yes

[ ] No

Question 3

Categorisation: Determine general algebraic expressions for integer patterns.

[Edexcel GCSE Nov2006-5H Q15a]

The table shows some rows of a number pattern.

Row 1	1^2	–	(0×2)
Row 2	2^2	–	(1×3)
Row 3	3^2	–	(2×4)
Row 4	4^2	–	(3×5)
Row *n*	.....		

In the table, write down an expression, in terms of n , for Row n .

.....

Question 4

Categorisation: Identify that $4n$ is a multiple of 4, $5n + 1$ is one more than a multiple of 5, and so on.

[Edexcel GCSE Nov2007-4I Q13 Edited]

Some expressions are listed below.
In each expression, n is a whole number.

Tick each of the three expressions which are always multiples of 9.

[ ] $3n^2$

[ ] $9n$

[ ] $3 \times 6n$

[ ] $3 + 6n$

[ ] $(3n)^2$

[ ] $n + 9$

Question 5

Categorisation: Use algebraic techniques to simplify an expression, potentially to yield properties about the resulting expression/number.

[Edexcel GCSE Nov2016-2H Q24 Edited]

Prove that, for all positive values of n

$$\frac{(n+2)^2 - (n+1)^2}{2n^2 + 3n}$$

is equal to

$$\frac{a}{b}$$

Where a and b are integers or variables to be found.

.....

Question 6

Categorisation: Prove that a given algebraic expression is a multiple of an integer (e.g. by writing in the form $4(\ldots)$ to demonstrate it is a multiple of 4).

[Edexcel GCSE June2009-4H Q22 Edited]

Prove that $(3n+1)^2-(3n-1)^2$ is a multiple of 4 , for all positive integer values of n .

.....

Question 7

Categorisation: Prove that a given algebraic expression is always an odd number (by writing in the form $2(\ldots)+1$).

[Edexcel GCSE June2007-5H Q24ii Edited]

n is a whole number.

Prove that $n^2+(n+1)^2$ is always an odd number.

.....

Question 8

Categorisation: Propose expressions for adjacent odd or even numbers, and combine in an addition to prove it is a multiple of a number.

[Edexcel GCSE Nov2008-4H Q20c Edited]

(c) Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6.

.....

Question 9

Categorisation: Use expressions for <u>different</u> odd/even integers (i.e. not necessarily consecutive, and by using separate variables, e.g. m and n) in more complicated expressions (e.g. involving squares), and prove it holds some numerical property.

[Edexcel GCSE Nov2005-5H Q23 Edited]

Prove algebraically that the sum of the squares of any two odd numbers leaves a remainder of 2 when divided by 4.

.....

Question 10

Categorisation: As above, but with <u>consecutive integers</u> (not consecutive odd/even) prove that resulting number is odd/even.

[Edexcel GCSE(9-1) Mock Set 2 Spring 2017 2H Q15]

Prove algebraically that the difference between the squares of any two consecutive integers is always an odd number.

.....

Question 11

Categorisation: As Question 9, but with consecutive odd/even integers.

[Edexcel GCSE(9-1) Mock Set 2 Spring 2017 1H Q19]

Prove that the sum of the squares of any three consecutive odd numbers is always 11 more than a multiple of 12

.....

Question 12

Categorisation: Same categorisation as Q9.

[Edexcel GCSE(9-1) Mock Set 1 Autumn 2016 - 1H Q13]

Prove algebraically that the difference between any two **different** odd numbers is an even number.

.....

Question 13

Categorisation: Proofs that involve combining the integers in two different ways.

[Edexcel GCSE March2013-1H Q21 Edited]

Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

.....

Question 14

Categorisation: Construct proofs involving the digits of a number, e.g. the number with the digits "abc" can be represented using $100a + 10b + c$ (e.g. as "254" $= 2 \times 100 + 5 \times 10 + 4$)

[Edexcel GCSE(9-1) Mock Set 2 Spring 2017 3H Q17a]

a , b , c are positive integers such that $a > b > c$

N is the largest three digit number that has the digits a , b , c. K is the smallest three digit number that has the digits a , b , c .

(a) Use algebra to show that the difference between N and K is always a multiple of 99

.....

Answers

Note: Some expressions will depend on the expressions used. e.g. If two consecutive integers are required, either $2n-1$ and $2n+1$ or $2n+1$ and $2n+3$ might be used. In the expressions below, the first odd integer is assumed to be $2n+1$.

Question 1

"odd"

Question 2

Yes

Question 3

$n^2-(n+1)(n-1)$

Question 4

"$9n$ ", "$3\times 6n$ " and "$(3n)^2$ "

Question 5

$a=1$, $b=n$

Question 6

$4(3n)$

Question 7

$2(n^2+n)+1$

Question 8

$6(n+1)$

Question 9

$4(m^2+n^2+m+n)+2$

Question 10

$2(x^2+x)+1$

Question 11

$12(x^2+3x+2)+11$

Question 12

$2(m - n)$

Question 13

$(n + 1)^2 - n^2$

Question 14

$99(a - c)$